Texas Instruments

TI-51-III

OWNER'S MANUAL

TEXAS INSTRUMENTS

KEY INDEX

This indexed keyboard provides a quick page reference to the description of each key

IMPORTANT

Record the serial number from the bottom of the unit and purchase date in the space below. The serial number is identified by the words "SERIAL NO." on the bottom case. Always reference this information in any correspondance

TI-61-III		
Model No.	Serial No.	Purchase Date

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Today, the way all of us deal with numbers and mathematics has been made easier by the hand-held calculator. A new speed, confidence, and accuracy are now possible in handling the "arithmetic" part of everyday life — for everyone! As hand-held calculators continue in their rapid evolution, higher power "advanced professional" machines — handling increasingly powerful mathematics — are becoming more and more available.

These machines bring a variety of powerful techniques right into the palm of your hand. Many techniques that previously required large volumes of tables, impossibly tedious calculations, or access to a large computer centre can now be carried out in a few keystrokes on a hand-held machine.

This book focuses on how today's advanced professional calculators (like the *Texas Instruments TI-51-III*) can make it easier than ever before for you to use some of the powerful tools of statistics and the mathematics of finance in your everyday and business decision-making. We'll focus on situations taken from the world of business and finance, and get together basic facts which will allow you to use calculator methods in arriving at more accurate and secure conclusions. We'll be concentrating on the *"how to use"* side of these techniques, stating them in straightforward, step-by-step, layman's language — including examples with keystroke solutions along the way. For those of you who want a brief look at the details and theory, we'll include a brief survey of some of the basics of statistics, too. Most importantly, this book is directed at being sure you get the most out of your calculator — to be sure you're fully aware of what it will (and will not) do for you.

"DEAR PIERRE,..." OR THE STORY OF STATISTICS

On some days it may seem that life itself (and business in particular) is just one big gamble. Appropriately enough the important science of statistics (as we now know it), traces its history to a gambler — a young nobleman from France. In 1654 Antoine Gombaud, having the title of Chevalier de Méré, was concerned over his luck at the gaming tables. He sought advice and counsel from the noted French mathematician, Blaise Pascal. Among the problems he put to Pascal was the question of how prize money should be divided among the players if a game is interrupted or "called off" for some reason.

This led Pascal into the study of probabilities — in particular he focused on the probability of one given player winning if a cancelled game were continued to completion. Pascal wrote a letter about these problems and his work on games of chance to another famous French mathematician, Pierre de Fermat. The resulting exchange of letters that followed was the begin-

ing of the evolving science of statistics -- whose methods are used in handling uncertain situations of all sorts today !

THE STORY OF CALCULATORS

Blaise Pascal was indeed an interesting and productive man -- for while he was busy giving birth to the science of probability and statistics, he was also tinkering with what became one of the world's first "calculating machines" by building on the ideas of men such as John Napier. Pascal's work in this area began the evolution of the mechanical calculator -- machines handling calculations rather slowly with the aid of complex entanglements of whirling gears,whizzing cranks, wheels, and windows. This evolution continued on up through 1890, when the punched card was pressed into data handling and calculating service in helping to take the 1890 U.S. Census. This led the way to later electric relay devices which continued to evolve into large-scale computers.

Then, a few years ago, people working in the electronics industry made several breakthroughs that resulted in the *integrated circuit*. Integrated circuits made it possible to process and store large amounts of information, in very small spaces, with little power and at low cost. These devices, coupled with the development of the inexpensive *"Light Emitting Diode"* (LED) display made hand-held calculators a reality. Recent advances in integrated circuits (IC's) are continuing to increase the amount of information storage and processing that can be handled on a single IC "chip". (The term IC "chip" refers to the tiny piece of silicon upon which an integrated circuit is fabricated.)

New highly flexible "chips" are making today's "Advanced Professional" and Programmable hand-held machines possible. With these advanced machines anyone -- almost anyplace -- can with the touch of a key execute a highly complex mathematical calculation rapidly and accurately.

THE "CALCULATOR DECISION-MAKING SOURCEBOOK"

Mathematics – including the mathematics of statistics and finance – is all around us and is part of many activities in our everyday and business lives. Your calculator can help handle the mathematical side of life quickly and accurately – without having to bother with lengthy computations. What's more, your advanced professional calculator can be a powerful ally as you handle decisions in your everyday and business life. This book has been designed to show you how.

What we've tried to do is put together a compact, accessible, step-by-step package of techniques enabling you to take a variety of decision-making situations and analyse them with keyboard solutions. This book was designed to work directly with your calculator, so be sure to use them together. Both of them have been designed for you.

An important first step is to get thoroughly acquainted with your calculator, to put it through its paces and to examine all aspects of its operation. *Chapter 1* of this book is a quick guided "tour" of all the features and keys of your calculator, along with brief examples illustrating the use of each feature. This "tour" is segmented into four major sections:

1) The keyboard basics
2) "Technical" functions and keys
3) Statistical functions and keys
4) Programming functions and keys

Touring in this fashion enables you to quickly get into the use of the *special powers* of your advanced machine – after briefly reviewing its basic operations.

The subsequent chapters in the book are packed with examples that illustrate how you can work with your machine in "Calculating Better Decisions". In each case a real life, business or financial situation is analysed for you.

Each example is broken down into the following segments — each identified with its own graphic symbol, as shown

Target : In each case the target is a brief statement of what types of calculation we'll be using to analyse the problem, and how to begin implementing the calculation.

Tools The formulae and facts needed to "calculate the decision" along with a very brief statement as to *why* each is used, *where* the techniques come from, and *how* they are tailored to the specific example.

Keying It In : Sample keystrokes to execute the solution (using the data given in the example), along with what you'll see in the display at key points in the calculation.

Decision Time : How to use the results of your calculation in arriving at a conclusion or decision.

Going Further : For some examples, a "going further" section is also included — discussing how additional information or conclusions may be drawn from the calculation you've just completed.

While you're busy using your calculator, don't forget that even though it may be packed with the latest in solid state technology — it still qualifies as a great toy — for children of all ages. Play with it ! Use it for exploring and "what iffing", as well as just idle doodling on the keys. You may just find yourself exploring patterns and relationships which can lead you to a new appreciation of the beautiful side of numbers and mathematics.

The TI-51-III you have just purchased is an advanced professional calculator designed specifically for those who demand a versatile and reliable business, scientific and mathematical tool. The availability of conversions, statistical analyses and a wide range of mathematical functions have been combined with the easy-to-use Algebraic Operating System to provide straightforward solutions to your most complex problems.

- **Algebraic Operating System (AOS)** allows you to enter mathematical expressions in the same order that they are algebraically stated. Parentheses, an integral part of AOS, ensure proper and accurate interpretation of expressions. Up to 9 parenthesis levels with 4 pending operations are available.
  Consider the expression
  $\frac{(3 \times 4 + 5 \times \tan 7^\circ)}{9^3} = 0.017303$ that can be
  entered directly as [(] 3 [×] 4 [+] 5 [×] 7 [tan] [)] [÷] 9 [y^x] 3 [=]
- **Complete Set of Mathematical Functions** including
  Arithmetic Functions with algebraic hierarchy
  Trigonometric Functions (including inverse functions)
  Angles measured in degrees, radians or grads
  Hyperbolic Functions (including inverse functions)
  Logarithmic Functions (both natural and common) with 10^x and e^x
  Factorial, Reciprocal, Percent and Change of Percent
  Square and Square Root, y^x and $\sqrt[x]{y}$
  Pi (π) accurate to 11 digits
  Constant feature for easy execution of repetitive calculations.
- **Addressable Memory System** with 10 separate memories for instant storage and recall of data. Complete memory arithmetic allows you to add, subtract, multiply or divide directly into any memory. Includes memory exchange with display
- **Linear Regression** routine provides both immediate statistical analysis of data and projection of new points. Trend-Line Analysis is also available.
- **Mean, Standard Deviation, Variance and Correlation** capabilities to analyse one or two-dimensional statistical data.
- **Totally Portable** when operating on its rechargeable battery system. It can also be operated while charging from an AC power source.

- **Conversions available from the keyboard provide easy transition between**
  - inches and millimetres
  - gallons (US) and litres
  - pounds (av) and kilograms
  - Fahrenheit and Celsius
  - degrees, radians and grads
  - polar and rectangular coordinates
  - degrees, minutes, seconds and decimal degrees
- **Complete Display Versatility, featuring :**
  - Standard 8-digit display
  - Scientific Notation entry from keyboard and automatically from calculations
  - Engineering Format displays scientific notation exponents as multiples of 3
  - Scientific or Engineering Notation removal
  - Fix Decimal control to select desired number of decimal places in the displayed number
  - Display value accuracy ensured by internal rounding
  - All results are calculated with 11 digits and rounded to obtain the displayed values
- **Automatic Clearing** - when the [=] key is pressed, all calculations are completed, the answer is displayed and the calculator is ready for the start of a new problem
- **Programmability** - 4 programming keys and 32 program steps are available for running "straight-line" programs
- **Automatic Power Saver** - Your calculator is designed to be energy efficient. After about 90 to 150 seconds of non-use, the display will shut down to a single decimal point travelling in the display. This keeps all of your current calculations "intact" while greatly decreasing the amount of power your machine consumes. Much less frequent battery recharges are required because of this built-in efficiency. To restore the display at any time, just proceed with a calculation, or press the [2nd] key twice.

The keys have been selectively positioned on the keyboard to provide for efficient calculator operation. Although many of the operations may be obvious, the following instructions and examples can help you develop skill and confidence in your solving routine.

INITIAL OPERATION

The fast-charge, nickal-cadmium battery pack furnished with your calculator was fully charged at the factory before shipping. However, due to shelf-life discharging, it may require charging before initial operation. If initially or during portable operation the display becomes dim or erratic, the battery pack needs to be charged.

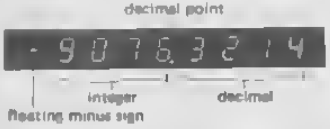
Under normal conditions, a fully charged battery pack provides typically 2-3 hours of continuous operation.

With the battery pack properly installed, charging is carried out by plugging the AC Adapter/Charger AC9900R into a convenient 220 V/50 Hz outlet and connecting the attached cord to the calculator socket. About 4 hours of charging restores full charge with the power switch off or 10 hours if the calculator is in use.
CAUTION : The battery pack will not charge if not properly installed in the calculator.

Sliding the ON/OFF switch to the right applies power to the calculator and sliding it to the left removes power. The power-on condition is indicated by a display which is alight.

STANDARD DISPLAY

In addition to power-on indication, the display provides numerical information complete with negative sign and decimal point and flashes on and off for an overflow, underflow or error condition. An entry can contain as many as 8 digits. All digits entered after the eighth are ignored.



Any negative number is displayed with a minus sign immediately to the left of the number.

See Appendix C for the accuracy of the displayed result.

DATA ENTRY KEYS

[0] through [9] **Digit Keys** - Enters the numbers 0 through 9.

[•] **Decimal Point Key** Enters Decimal Point. The decimal point can be entered wherever needed. If no decimal point is entered, it is assumed to be to the right of the number, and will appear when a non-number key is pressed. A zero will precede the decimal point for numbers less than 1 unless all ten available display digits are used. Trailing zeros on the decimal portion of a number are not normally displayed. Only the first decimal point entered is accepted, all others are ignored

[2nd] [π] **Pi Key** - Enters the value of pi (π) to 11 significant digits (3.1415926536) for calculations, display indicates the rounded value to 8 places

[EE] **Enter Exponent Key** - Instructs the calculator that the subsequent number entry is an exponent of 10. After the [EE] key is pressed all further results are displayed in scientific notation format until [CLR] or [2nd] [CA] is pressed or until the calculator is turned off. [INV] [EE] or [INV] [2nd] [ENG] can remove this format if the displayed number is in the range $\pm\ 9.9999999 \times 10^{7}$ to $\pm\ 1 \times 10^{-7}$.

[+/−] **Change Sign Key** - Instructs the calculator to change the sign of the displayed number. When pressed after [EE], changes sign of the exponent.

Example : Enter −12.6

Press	Display/Comments
12.6 [+/−]	−12.6

CLEARING OPERATIONS

[CE] **Clear Entry Key** - Clears entries made with the digit, decimal point and change-sign keys when pressed before a function key. This key does not clear calculated results, numbers recalled from memory or π. [CE] also stops the flashing of the display when needed. Use of this key does not affect pending operations.

[CLR] **Clear Key** Clears calculations in progress, the constant and the display. It resets scientific notation to standard format and will stop a flashing display. This key does not affect the contents of user memories, program memory, fixed-point (fix-decimal) location, angular mode or engineering format.

[2nd] [CA] **Clear All Key** - Clears the display, all memories including program memory, the constant and calculations in progress. Restores standard display mode and resets angular mode to degrees. Eliminates fixed-point (fix-decimal) format.

The calculator effectively clears itself after most calculations. When the [=] key is pressed to complete a calculation, the answer is displayed and the calculator is ready for the start of a new problem without pressing any of the clear keys. The contents of the user memories are not automatically cleared

DUAL FUNCTION KEYS (2nd and INV)

Most of your calculator's keys have dual functions. The first function is printed on the key and the second function is written above it. To execute a function shown on a key, simply press the desired key. To use the second function of a key, press the 2nd key, then press the key immediately below the desired second function. For example, to find the natural logarithm of a number, press ln x . To find the common logarithm of a number, press 2nd ln x . In order to make sequences of this type clear, in this manual it will be shown as 2nd log . First function operations, therefore, are indicated by [ ] . Second functions are indicated by [ ]. When 2nd is pressed twice in succession or if a key that does not have a second function is pressed, the calculator returns to first function operation.

The inverse key INV provides additional computing capabilities without increasing the number of keys on the keyboard just like the 2nd key does. When INV precedes another key, the purpose of that key is reversed. The inverse can be used with the following keys to obtain the indicated function.

1st function keys	2nd function keys
sin → $\sin^{-1}$	sinh → $\sinh^{-1}$
cos → $\cos^{-1}$	cosh → $\cosh^{-1}$
tan → $\tan^{-1}$	tanh → $\tanh^{-1}$
SUM → subtract	PROD → divide
EE → removes EE	ENG → removes ENG or EE
	FIX → removes FIX
	conversions → reverses conversions
	Mean → Mean of x data
	Var → Variance of x data
	S. Dev → Standard Deviation of x data

This key can also be used to obtain the mean, standard deviation and variance of the independent variable, x, in the linear regression routine. An inverse instruction may be cancelled by pressing INV a second time, if no other keys have been pressed, or by pressing a key without an inverse function. When used in conjunction with the second function key, the inverse key can be pressed before or after the second function key is pressed, i.e. INV 2nd, [ ] or 2nd INV [ ]. When programming, the INV key must always precede the 2nd key.

For examples of INV uses with a specific key, see the section relating to each key.

DISPLAY FORMATS

Even though a maximum of 8 digits can be entered or displayed, the internal display register always retains results to 11 digits. The results are then rounded for display only.

In addition to the versatile 8-digit standard display, there are several other display capabilities that increase the operating range and flexibility of your calculator.

Scientific Notation

Any number can be entered as the product of a value (mantissa) and 10 raised to some power (exponent).

This capability allows you to work with numbers as small as $\pm 1 \times 10^{-99}$ or as large as $\pm 9.9999999 \times 10^{99}$. Numbers smaller than $\pm 1 \times 10^{-7}$ or larger than $\pm 9.9999999 \times 10^{7}$ must be entered in scientific notation. When calculations exceeds these limits, the calculator automatically shifts into scientific notation. The entry procedure is to key in the mantissa (including its sign), then press EE and enter the exponent of 10 and its sign.

For example, the number 320,000,000,000 can be written as 3.2×10^{11} and can be entered into the calculator as:

Press	Display/Comments
CLR	0
3 2	3.2
EE	3.2 00
1 1	3.2 11

More than 2 digits can be entered after pressing EE, but only the last two entered are retained as the exponent.

In scientific notation, a positive exponent indicates how many places the decimal point of the mantissa should be shifted to the right. If the exponent is negative, the decimal should be moved to the left.

Regardless of how a mantissa is entered for scientific notation, the calculator *normalises* the number, displaying a single digit to the left of the decimal point, when any function or operation key is pressed.

Example Enter 6025×10^{20}

Press	Display/Comments
CLR	0
6025	6025
EE	6025 00
20	6025 20
+	6.025 23

A mantissa resulting from a calculation is displayed to 8 digits, but internally is carried to 11 digits. This 11-digit value is the one used for all ensuing calculations. See Appendix C.

The change sign key can be used to attach a negative sign to the mantissa and to the power-of-ten exponent. Simply press +/− after entry of the mantissa to change its sign or after the exponent to change its sign. To change the sign of the mantissa or to enter numbers in its decimal portion after the EE key has been pressed, press . , then enter the mantissa's sign or additional numbers to the decimal portion.

Example : Enter -4.962×10^{-13} then complete the decimal portion of the mantissa to read -4.96236×10^{-13}.

Press	Display/Comments	
CLR	0	
4.962 +/−	−4.962	Enter mantissa and sign
EE	−4.962 00	
12 +/−	−4.962 12	Enter exponent and sign
+/−	−4.962 12	Change exponent sign
+/−	−4.962-12	Change exponent sign again
. +/−	4.962-12	Change mantissa sign
36 +/−	−4.96236-12	Complete the mantissa

Data in scientific notation form can be intermixed with data in standard form. The calculator converts the entered data for proper calculation. After the EE key has been pressed the calculator displays all results in scientific notation format until CLR , 2nd CA , INV EE or INV 2nd ENG is pressed, or until the calculator is turned off.

Example : $1.816 \times 10^{3} - 581.43219 = 1.2345678 \times 10^{5} = 1234.567809$.

Press	Display/Comments
CLR	0
1.818 EE	1.818 00
3 +/−	1.818 03
581.432191 =	1.2345678 03
INV EE	1234.5678

When INV EE is pressed to remove scientific notation and the number is outside of the range $9.9999999 \pm 1 \times 10^{7}$ to $\pm 1 \times 10^{7}$, the calculator will return to standard format only when or if a calculated result is in the displayable range

Example : $(7 \times 10^{7} + 5 \times 10^{10}) \div 25 - 25 = 80112000$.

Press	Display/Comments
7 EE	7 00
7 +	7.07
5 EE	5 00
10 = INV EE +	5.007 10 8.007 10
25 = +	2.0028 09
25 =	80112000.

Engineering Notation

This modified form of scientific notation is accessed by pressing 2nd ENG The displayed value in this mode consists of a mantissa and an exponent that have been adjusted so that the exponent is a multiple of three (10^{12}, 10^{-6}, etc.) and the mantissa has 1, 2 or 3 digits to the left of the decimal point. This allows the calculator to display results in units that are readily usable such as 10^{-12} for millimetres, 10^{6} for megohms or 10^{-9} for nanoseconds.

Example : What is the diameter of a cable in microns (1 micron = 10^{-6} metre) whose circumference is 3×10^{-3} metres ?

$C = \pi d$ $d = C/\pi$

Press	Display/Comments
2nd ENG	0. 00
3 EE	3 00
3 +/− +	3.-03
2nd π =	954.92966 -06

Pressing INV 2nd ENG or 2nd CA will remove this display mode, CLR or INV EE does not affect it

Fix-Decimal Control

In standard display format, scientific notation or engineering notation, you can selectively choose the number of digits to display following the decimal point. Pressing 2nd fix, then entering the desired number of decimal places (0 to 7), instructs the calculator to round all results to the selected number of decimal places.

Pressing 2nd CA, 2nd fix 8, 2nd fix 9, or INV 2nd fix returns the calculator to the standard display. Data entries can still be made with 8 digits with all subsequent calculations using the 11-digit unrounded results. Only the display is altered to the requested number of decimal places.

Example : 2/3 = 0.6666667

Press	Display/Comments
2 ÷	2.
3 =	0.6666667
2nd fix 5	0.66667
2nd fix 2	0.67
2nd fix 0	1.

Remember that the display value is *rounded* to the desired format.

Example : $1 \times 10^{-3} \div 2 = .0005$

Press	Display/Comments
2nd CA	0
1 EE	1 00
3 +/− ÷	1.-03
2 =	5.-04
2nd fix 2	5.00 -04
INV EE	0. 00
2nd fix 3	0.001
2nd fix 4	0.0005
2nd fix 5	0.00050

Flashing Display

The display flashes off and on whenever the limits of the calculator are violated or when an improper mathematical operation is requested. Press CE to stop the flashing without disturbing any calculations in progress. Calculations can continue from this point if the number in the display is still usable. See Appendix B for a complete list of error and overflow/underflow conditions.

The Algebraic Operating System's method of entering numbers and operations is straightforward allowing entry of most problems just as they are mathematically stated. The accuracy of results is discussed in Appendix C.

BASIC KEYS

[+] , [−] **Add and Subtract Keys** - Correspondingly alters the present display value by the next entered quantity. These keys also complete any previously entered arithmetic (+, −, x, ÷), y^x, $\sqrt[x]{y}$ or Δ% functions.

[x] , [÷] **Multiply and Divide Keys** - Correspondingly alters the present display value by the next entered quantity. These keys also complete any previously entered multiply, divide, y^x, $\sqrt[x]{y}$ or Δ% functions.

[=] **Equals Key** - Computes results by completing all previously entered numbers with associated operations, preparing the calculator for a new problem.

[x≷y] **x Exchange y Key** - Exchanges factors in multiplication and exchanges divisor and dividend in division. Interchanges x and y in Δ%, y^x and $\sqrt[x]{y}$ calculations. Also used for data entry and result display for polar to rectangular conversions and linear regression problems.

Pressing [CLR] at the beginning of a new sequence clears any calculations in progress and always ensures that no pending operations from prior calculations remain. This is not required if the previous problem used [=] to obtain the result. Following [=] with a numeric entry accomplishes the same as pressing [CLR] , except that [=] does not remove scientific notation or stop a flashing display or clear the constant.

Pressing any two of the operations keys (+, −, x, ÷, y^x, $\sqrt[x]{y}$, and Δ%) in succession causes a flashing display. Also, following any of these with = or), or preceding with (, causes the same result.

Example . 23.79 + .64 − 6 = 18.33

Press	Display/Comments
[CLR]	0
23.79 [+]	23.79
.64 [−]	24.33
6 [=]	18.33

Again note that the numbers and functions are entered in the same order as they are mathematically stated.

Example : − 4 x 7.3 ÷ 2 = − 14.6

Press	Display/Comments
4 [+/−] [x]	− 4.
7.3 [+]	− 29.2
2 [=]	− 14.6

COMBINING OPERATIONS

After a result is obtained in one calculation it may be directly used as the first number in a second calculation. There is no need to re-enter the number from the keyboard.

Example
1.84 + 0.39 = 2.23 then (1.84 + 0.39)/365 = 0.0061096.

Press	Display/Comments
1.84 [+]	1.84
.39 [=]	2.23 1.84 + 0.39
[+]	2.23
365 [=]	0.0061096 2.23 ÷ 365

HIERARCHY OF OPERATIONS

Algebraic hierarchy is an essential feature of the Algebraic Operating System. To efficiently combine operations, the standard rules of algebraic hierarchy have been specifically programmed into the calculator.

These algebraic rules assign priorities to the various mathematical operations. Without a fixed set of rules, expressions such as 5 x 4 + 3 x 2 could have several meanings

5 x (4 + 3) x 2 = 70
or 5 x 4 + 3 x 2 = 26
or (5 x 4 + 3) x 2 = 46
or 5 x (4 + 3 x 2) = 50

Algebraic hierarchy rules state that multiplication is to be performed before addition. So algebraically, the correct answer is (5 x 4) + (3 x 2) = 26. The complete list of priorities for interpreting expressions is :

1) Special Functions
2) Percent Change (Δ%)
3) Exponentiation (y^x), Roots ($\sqrt[x]{y}$)

4) Multiplication, Division
5) Addition, Subtraction
6) Equals

1) Special functions (trigonometric and hyperbolic, logarithmic, square, square root, factorial, e^x, 10^x, percent, reciprocal and conversions) immediately replace the displayed value with its functional value.

2) Percent change has only the ability to complete other percent change operations

3) Exponentiation (y^x) and roots ($\sqrt[x]{y}$) are performed as soon as the special functions and percent change are completed.

4) Multiplication and division are performed after completing special functions, percent change exponentiation, root extraction and other multiplication and division.

5) Addition and subtraction are performed only after completing all operations through multiplication and division as well as other addition and subtraction.

6) Equals completes all operations.

Operations of the same priority are performed left to right.

To illustrate, consider the interpretative order of the following example .

Example : $4 - 5^2 \times 7 + 3 \times 0.5^{\cos 60} = 3.2413203$

Press	Display/Comments	
4 [÷]	4.	(4÷) is stored
5 [x^2]	25.	(5^2) special function evaluated immediately
[×]	0.16	($4 \div 5^2$) ÷ evaluated because × is same priority as ÷,
7 [+]	1.12	× higher priority then + so ($4 - 5^2 \times 7$) evaluated, + stored
3 [×]	3.	(3×) stored
.5 [y^x]	0.5	.5 entered, y^x stored
60 [COS]	0.5	Cos 60° evaluated immediately.
[=]	3.2413203	Completes all operations $5^{\cos 60}$ evaluated, then $3 \times 5^{\cos 60}$ next, then this is added to 1.12.

Thus, by entering the expression just as it is written, the calculator correctly interprets it as :

$\{[(4 - 5^2) \times 7] + (3 \times 0.5^{\cos 60})\}$

The important things to remember here are that operations are enacted strictly according to their relative priority as stated in the rules. The calculator remembers all stored operations and recalls each and its associated number for execution at exactly the correct time and place. Once familiar with the order of these operations, you will find most problems are extremely easy to solve because of the straightforward manner in which they can be entered into the calculator. Additional control over the order of interpretation is provided through the use of parentheses.

PARENTHESES

There are sequences of operations for which you must instruct the calculator exactly how to evaluate the problem and produce the correct answer. For example

$$4 \times (5 + 9) \div (7 - 4)^{(2+3)} = ?$$

To evaluate this expression as written using only the calculator hierarchy, many independent steps would be required. Also, intermediate results would have to be stored and the sequence certainly could not be input in the same order in which it is written.

Parentheses should be used here and whenever a mathematical sequence cannot be directly entered using the previously mentioned algebraic rules or to simplify entry of a problem without reference to the hierarchy rules.

To illustrate the benefit of parentheses, try the following experiment. Press (5 x 7), and you will see the value 35 displayed. The calculator has evaluated 5 x 7 and replaced it with 35 even though the [=] was not pressed. Because of this function of parentheses, the algebraic rules now apply their hierarchy of operations to each set of parentheses. Use of parentheses ensures that your problem can be keyed in just as you have written it down. The calculator remembers each operation and evaluates each part of the expression as soon as all necessary information is available. When a closed parenthesis is encountered, all operations back to the corresponding open parenthesis are completed.

Example : $4 \times (5 + 9) \div (7 - 4)^{(2+3)} = 0.2304527$

Key in this expression and follow the path to completion.

Press	Display/Comments	
4 x (	4.	(4x) stored pending evaluation of parentheses
5 +	5.	(5+) stored
9)	14.	(5 + 9) evaluated
+	56.	Hierarchy evaluates 4 x 14
)	56.	56 – stored pending evaluation of parentheses
7 –	7.	(7–) stored
4)	3.	(7 – 4) evaluated
y^x (	3.	Prepares for exponent
2 +	2.	
3)	5.	(2 + 3) evaluated
=	0.2304527	$(7 - 4)^{(2+3)}$ evaluated then it is divided into 4 x (5 + 9)

There are limits on how many operations and associated numbers can be stored. Actually as many as nine parentheses can be open at any one time and four operations can be pending, but only in the most complex situations would this limit be approached. If you do attempt to open more than 9 parantheses or if the calculator tries to store more than four operations, the display flashes

Example : 5 + {8/[9 – (2/3)]} = 5.96

Press	Display/Comments	
5 + (	5.	
8 + (	8.	
9 – (	9.	
2 +	2.	
3)	0.6666667	(2/3) evaluated
)	8.3333333	[9 – (2/3)] evaluated
)	0.96	{8/[9 – (2/3)]}
=	5.96	5 + {8/[9 – (2/3)]}

Because the = key has the capability to complete all pending operations whenever it is used, it could have been used here instead of the) keys. Try working this problem again and pressing = instead of the first)

Exemple : $3 \times \{4^{12(-\sqrt[4]{7})}\} = 4.7000434$

Press		Display/Comments	
CLR	(	0.	
3	× (	3.	
4	y^x (	4.	
2	y^x (	2.	
7	$\sqrt[x]{y}$	7.	
4	)	1.6285766	$\sqrt[4]{7}$
+/−		− 1.6285766	$-(\sqrt[4]{7})$
)		0.3238558	$2(\sqrt[4]{7})$
(		1.5666811	4^{323-}
)		4.7000434	$3 \times 4^{333-}$

Each time a closed parenthesis is encountered, the contents are evaluated back to the nearest open parenthesis and are replaced with a single value. Knowing this you can structure the order of interpretation for whatever purpose you may want. Specifically, you can check intermediate results.

The simplest operations to describe and understand are the single-variable functions. These functions operate on the displayed value immediately, replacing the displayed value with its corresponding function value. These functions do not interfere with any calculations in progress and can therefore be used at any point in a calculation. Be sure that each calculation has been completed before the next key is pressed. Key entries are not recognised while a calculation is being performed.

RECIPROCAL AND FACTORIAL

[1/x] **Reciprocal Key** - Calculates the reciprocal of the value, x, in the display register by dividing x into 1. $x \neq 0$.

[2nd] [x!] **Factorial Key** - Calculates the factorial ($1 \times 2 \times 3 \times 4 \times \quad \times x$) of the value, x in the display for integers $0 \leqslant x \leqslant 69$ $0! = 1$ by definition

Example : $1/3.2 = 0.3125$

Press	Display/Comments
3.2 [1/x]	0.3125

Example : $1/(-12 + 5!) = 0.0092593$

Press	Display/Comments
12 [+/-] [+]	− 12.
5 [2nd] [x!]	120.
[=]	108
[1/x]	0.0092593

Note that as soon as one of the math function keys is pressed, the displayed value is immediately replaced with its corresponding function value

LOGARITHMS

[ln x] **Natural Logarithm Key** - Calculates the natural logarithm (base e) of the value, x, in the display register $x > 0$.

[2nd] [log] **Common Logarithm Key** - Calculates the common logarithm (base 10) of the value, x, in the display register. $x > 0$.

Example : Log (1 + ln 1.7) = 0.1848697

Press		Display/Comments
[(]		
1	[+]	1.
1.7	[ln x]	0.5306283
[)]		1.5306283
[2nd]	[log]	0.1848697

POWERS OF 10 AND e

[e^x] **e to the x Power Key** - Calculates the natural antilogarithm of the value, x, in the display register. $-227.95592 \leq x \leq 230.25850$.

[2nd] [10^x] **10 to the x Power Key** - Calculates the common antilogarithm of the value, x, in the display register. $-99 \leq x < 99\ 999999$

Example : $e^{(3+10^{0.3})} = 147\ 71169$

Press		Display/Comments
[(]		0.
3	[+]	3.
.3	[2nd] [10^x]	1.9952623
[)]		4.9952623
[e^x]		147.71169

ANGLE CALCULATIONS

Your calculator provides maximum flexibility when performing calculations involving angles.

Angular Modes

Angles can be measured in degrees, radians or grads (right angle = $90^\circ = \pi/2$ radians = 100 grads). You select the mode desired by pressing either [2nd] [Deg], [2nd] [Rad] or [2nd] [Grad]. The calculator powers-up in the degree mode and stays in that mode until altered by one of the other choices. Once in a certain angular mode, all entered and calculated angles are measured in the units of that mode until another mode is selected, [2nd] [CA] is pressed or until the calculator is turned off. [2nd] [CA] restores the degree mode. [CE] and [CLR] do not effect the angular mode.

The angular mode has absolutely no effect on calculations unless the trigonometric functions or polar to rectangular conversions are being performed. Selecting the angular mode is easy to do — and easy to forget. *Neglecting this step is responsible for a large number of errors in operating any calculation device that offers a choice of angular units.*

TRIGONOMETRIC FUNCTIONS

[sin] , [cos] , [tan] **Trigonometric Keys** - Calculates the sine, cosine or tangent of the value in the display register

Example : sin 30° + tan 315° = – 0,5

Press	Display/Comments
[2nd] [CL]	0.
30 [sin] [+]	0.5
315 [tan]	– 1.
[=]	– 0.5

Trigonometric values can be calculated for angles greater than one revolution. See page 164 for additional information.

HYPERBOLIC FUNCTIONS

[2nd] [sinh] , [2nd] [cosh] , [2nd] [tanh] **Hyperbolic Function Keys** - Calculates the hyperbolic sine, cosine or tangent of the value x in the display register.
$|x| \leq 227.95592$ for sinh and cosh.
$-227.95592 \leq x \leq 230.25850$, $x \leq \pm 227.95592$ for sinh and cosh.

Enter	Display/Comments
100 [+]	100
3.3 [EE] 2 [=]	3.030303-01
[2nd] [tanh]	2.940833-01

INVERSE TRIGONOMETRIC AND HYPERBOLIC FUNCTIONS

[INV] **Inverse Key** - Preceding another key, reverses the intention of that key. When used with the trig or hyperbolic functions, the inverse of those functions is obtained For example, arcsine ($\sin^{-1}$) is obtained by pressing [INV] [sin] , hyperbolic arctangent ($\tanh^{-1}$) results from [INV] [2nd] [tanh]

The inverse trig functions calculate the angle whose functional value is in the display. The largest angle resulting from an arc function is 180 degrees (π radians or 200 grads) Because these functions have many angle equivalents, i.e., arcsin 5 = 30°, 150°, 390°, etc., the angle returned by each function is restricted as follows :

Arc Function	Range of Resultant Angle
arcsin x ($\sin^{-1}$ x)	0 to 90°, π/2 radians, or 100 G
arcsin − x ($\sin^{-1}$ − x)	0 to −90°, − π/2 radians, or − 100 G
arccos x ($\cos^{-1}$ x)	0 to 90°, π/2 radians, 100 G
arccos − x ($\cos^{-1}$ − x)	90° to 180°, π/2 to π radians, or 100 to 200 G
arctan x ($\tan^{-1}$ x)	0 to 90°, π/2 radians, or 100 G
arctan −x ($\tan^{-1}$ − x)	0 to − 90°, − π/2 radians, or − 100 G

For arcsin x and arccosin x, $-1 \leqslant x \leqslant 1$.

Example : $\pi/4 + \tan^{-1}(.2\pi) = 1.3463803$

Press	Display/Comments
2nd Rad	0.
2nd π +	3.1415927
4 + (	0.7853982
.2 × 2nd π)	0.6283185
INV tan	0.5609821
=	1.3463803

The selection of the radian mode could have been made at any point before INV tan

Note the following restrictions.

arcsinh x ($\sinh^{-1}$ x)	$-10^{50} < x < 10^{50}$
arccosh x ($\cosh^{-1}$ x)	$1 \leqslant x < 10^{50}$
arctanh x ($\tanh^{-1}$ x)	$-1 < x < 1$

Example : $.25 + \tanh^{-1}(.866) = 1.5669563$.

Enter	Display/Comments
.25 +	0.25
.866 INV 2nd tanh	1.3169563
=	1.5669563

SQUARE AND SQUARE ROOT

x^2 **Square Key** - Calculates the square of the number in the display register, $10^{-50} < |x| < 10^{50}$.

$\sqrt{x}$ **Square Root Key** - Calculates the square root of the number in the display register, $x \geq 0$.

Example : $[\sqrt{3.1452} - 7 + (3.2)^2]^{12} = 2.239078197$

Press		Display/Comments
3.1452	[$\sqrt{x}$	1.7734712
7	+	− 5.2265288
3.2	x^2	10.24
]		5.0134712
$\sqrt{x}$		2.2390782

UNIVERSAL ROOTS AND POWERS

y^x **Universal Power Key** - Raises the display register value, y to the x power. The entry sequence is y y^x x followed by an operation key or equal. $y \geq 0$.

$\sqrt[x]{y}$ **Universal Root Key** - Takes the x root of the value, y, in the display register. The entry sequence is y $\sqrt[x]{y}$ x followed by an operation key or equals, $y \geq 0$, $x \neq 0$.

$x \leftrightarrow y$ **x Exchange y Key** - Interchanges the x and y values after they have been keyed in. Can also be used with arithmetic operations and special calculations.

These maths functions do not act on the display register immediately. They require entry of a second value followed by an operation before the function can be realized.

Example : $\sqrt[3]{2.36^{-23}} = .9362893421$

Press		Display/Comments
2.36	y^x	2.36 Enter y for y^x
.23	+/−	− 0.23 Enter x for y^x
$\sqrt[x]{y}$		0.8207866 Produces y for $\sqrt[x]{y}$
3	=	0.9362893 Enter x for $\sqrt[x]{y}$ and produce answer

Note that logarithms are used in computing universal powers and roots. Therefore, a few entries involving negative numbers, zero and one are invalid and will produce a flashing display. For example, any negative y value will cause a flashing display.

PERCENT AND CHANGE PERCENT

[%] **Percent Key** - Converts the displayed number from a percentage to a decimal

[2nd] [Δ%] **Percent Change Key** - Calculates the percentage change between two values. Press x_1 [2nd] [Δ%] x_2 [=] and $\frac{x_1 - x_2}{x_2} \times 100$ is calculated

Example : 43.6 % = 0.436.

Press	Display/Comments
43.6 [%]	0.436

Example What is the percentage increase (markup) of a £766.48 suits of furniture which wholesales for £515.22 ?

Press	Display/Comments
766.48 [2nd] [Δ%]	766.48
515.22 [=]	48.767517

The suits has been marked up almost 49 %.

When [%] is pressed after an arithmetic operation, add-on, discount and percentage can be computed,

[+], n [%] [=] adds n% to the displayed value

Example : What is the total cost of a £45 overcoat when there is a 5 % sales tax ?

Press	Display/Comments
45 [+]	45.
5 [%]	2.25
[=]	47.25

Note that the percent (tax) is shown for recording, if necessary, then the total is displayed.

[−] n [%] [=]	subtracts n% from the displayed value

Example : How much do you have to pay for a £110 television that has been discounted 15 % with 6 % sales tax ?

Press	Display/Comments	
110 [−]	110.	Enter amount
15 [%]	16.5	15 % of 110
[+]	93.5	110 - 15 %
6 [%]	5.61	6 % of 93.50
[=]	99.11	Total Cost

[×] n [%] [=]	multiplies the displayed value by n%

Example : If you have hiked 35 % of a 62-mile trail, how far have you traveled ? In other words, what is 35 % of 62 ?

Press	Display/Comments
62 [×]	62.
35 [%]	0.35
[=]	21.7

You have traveled 21.7 miles.

[÷] n [%] [=]	divides the displayed value by n%

Example . If you have eaten 9 meals and find that 30 % of your food supply is gone, how many meals will your initial food supply provide ? 9 is 30 % of what number ?

Press	Display/Comments
9 [÷]	9.
30 [%]	0.3
[=]	30

Your initial food supply will provide for 30 meals.

Your calculator has ten user-accessible memories to greatly increase the flexibility of calculations. Because there are ten memories, you must specify which memory you are addressing by entering its number, n = 0 through 9 immediately after pressing any memory related key. Failure to enter one of these numbers after a memory key results in a flashing of the current display value. These memory registers can store or accumulate data for later use, in a variety of ways.

STORING AND RECALLING DATA

STO n Store Key - Stores the display value into memory register n, n = 0 through 9. Any previously stored data in n is cleared.

RCL n Recall Key - Recalls and displays the value stored in memory register n and retains the value in memory. A recalled number can be used as a number entry in any mathematical expression, n = 0 through 9.

Example. Store and recall 3.012 in memory 2.

Press	Display/Comments
3.012 STO 2	3.012
CLR	0
RCL 2	3.012

Use of these keys can save you keystrokes by storing long numbers that are to be used several times.

Example: Evaluate $3x^3 - x - 7.1$ for $x = 2.9467281$

Press	Display/Comments
CLR	0.
3 ×	3.
2.9467281 STO 1	2.9467281
x^3	6.6832065
−	26.049616
RCL 1	2.9467281
−	23.10289
7.1 =	16.002891

The long value of x only had to be entered once. The storage and recall did not interfere with calculator operations.

The memories can also be used to hold intermediate results as well as repetitive numbers.

Example : Evaluate $\frac{[\sin(3x/2) - \cos(3x/2)]}{x}$

for x = 20.682177 degrees.

Press	Display/Comments	
2nd CA ( ) ( )	0.	
3 x	3.	
20.68277 STO 1 +	62.048531	Store x
2) STO 2	31.023266	Store 3x/2
sin −	0.8153961	
RCL 2	31.023266	Recall x
cos	0.8569681	Cos(3x/2)
) +	−0.341572	
RCL 1	20.682177	Recall x
=	−0.0165153	Answer

MEMORY ORGANISATION

Because of the complexity of some of the statistical calculations, the calculator preempts certain memories to store data and results for these advanced computations. Also, memories 8 and 9 are used for storing program steps 17 through 31 (steps 17-24 in memory 9, steps 25-31 in memory 8). The chart below shows the arrangement and use of the calculator's 10 memories.

Memory Number

0	1	2	3	4	5	6	7	8	9
	STATISTICAL CALCULATIONS							PROGRAMMING	

Cleared By 2nd CA

DIRECT REGISTER ARITHMETIC

You can store a displayed number at any time during a calculation without affecting the calculation in any way. Additionally, you can add, subtract, multiply and divide the displayed value for calculations in progress. Pressing 2nd CA clears the memories as well as the entire calculator

SUM **n Sum Key** - Adds the displayed value to the content of memory register n and stores the result in n. n = 0 through 9.

INV SUM **n Subtract Sequence** - Subtracts the displayed value from the content of memory register n and stores the result in n, n = 0 through 9.

2nd PROD **n Product Key** - Multiplies the content of memory register n by the displayed value and stores this product in n, n = 0 through 9.

INV 2nd PROD **n Divide Key** - Divides the content of memory register n by the displayed value and stores the result in n, n = 0 through 9.

These capabilities eliminate the lengthy recall, perform operation, store-again sequences.

Example . Evaluate $x^2 + 9$ for x = −1, 2, 3 and total the results.

Press	Display/Memory 3
1 +/− x^2 +	1. 0
9 = STO 3	10. 10
2 x^2 +	4. 10
9 = SUM 3	13. 23
3 x^2 +	9. 23
9 = SUM 3	19. 41
RCL 3	41. 41

Notice that the first evaluation was placed in memory 3 using the STO key. The STO clears any previous content of that register before storing the new value

Example . The percentage of students completing each year at a particular college is 76,8 % first year, 81,3 % second year, 92,2 % third year and 95,9 % last year. What percentage of the students graduate and what percentage complete their third and fourth years ?

Press	Display/Comments
76,8 % X	0.768
81,3 % X	0.624384
92,2 % STO 1 X	0.575682
95,9 % 2nd PROD 1 =	0.552079
RCL 1	0.884198

About 55 % of the students that enter the school graduate. Over 88 % of those entering their junior year graduate

MEMORY/DISPLAY EXCHANGE

2nd EXC **n Exchange Key** - Exchange the content of memory register n with the display. The display value is stored and the previously stored value is displayed.

The exchange key has several uses. You can use it to examine two results without losing either. Also, numbers can be temporarily stored and used as needed.

Example . Evaluate $A^2 + 2AB + B^2$ for A = 0.268963 and B = 1.255632.

Press	Display/Comments	
.258963 STO 1 x^2 +	0.0670618	Store A
1.255632 X	1.255632	Enter B
2nd EXC 1	0.258963	Store B, Recall A
X	0.3251622	A x B
2 +	0.7173863	$A^2 + 2AB$
RCL 1	1.255632	Recall B
x^2	1.5766117	B^2
=	2.293998	Answer

There are several often-used mathematical sequences that have been programmed into your calculator. These operations have been specially designed to provide optimum calculator efficiency by minimising the number of keystrokes required to execute these iterative sequences.

CALCULATIONS WITH A CONSTANT

2nd CONST Constant Key - Stores a number and an operation for use in repetitive calculations. Used with the +, −, x, ÷, y^x, $\sqrt[x]{y}$ and Δ% operations.

The entry sequence is the same for all operations - enter the operation, then the repetitive number, m, followed by 2nd CONST. After the constant is stored, additional calculations are completed by entering the variable and pressing =.

+ | m 2nd CONST adds m to each subsequent entry.
− | m 2nd CONST subtracts m from each subsequent entry.
x m 2nd CONST multiplies each subsequent by m.
÷ m 2nd CONST divides each subsequent entry by m
y^x m 2nd CONST raises each subsequent entry to the m power i.e. y^m
$\sqrt[x]{y}$ m 2nd CONST takes the mth root of each subsequent entry, i.e. $\sqrt[m]{y}$.
m 2nd Δ% 2nd CONST calculates the percentage change between each subsequent entry and m

$$\text{by } \frac{x_1 - m}{m} \times 100$$

Performing statistical calculations (linear regression, mean, standard deviation, etc.), pressing CLR or 2nd CA or entering any of the above arithmetic operations removes or changes the constant.

Note in the following example that the constant can be entered as part of a normal problem sequence.

Example . Divide .02, tan 22°, 2 x 10^{22} and $(2222)^3$ by .89

Press		Display/Comments
2nd Deg		0
02	+	0 02
89	2nd CONST =	0.0224719
22	tan =	0.4539621
2	EE	2 00
22	=	2 247191 22
2222	x^3 =	5.5475101 06

During these calculations you can use any of the maths functions, select a fixed decimal point, use memory operations and conversions or vary the display format.

UNIT CONVERSIONS

A selected number of conversions is available directly from the keyboard. These are accessed by entering the number to be converted, then pressing 2nd followed by the desired conversion. Conversions can be made between the following quantities.

Degrees, minutes, seconds (DDD.mmss)	and	Decimal Degrees (DDD.dd)
Fahrenheit	and	Celsius (Centigrade)
Degrees	and	Radians
Grads	and	Radians
Inches	and	Millimetres
Gallons (U.S.)	and	Litres
Pounds (av)	and	Kilograms

The INV key can be used to reverse the effect of the conversions as listed on the keyboard. Conversions between degrees, minutes and seconds and decimal degrees is based on the relationship of degrees in decimal (DD.dd) = Integer degrees (DD) + minutes (mm)/60 + seconds (ss)/3600. Minutes and seconds must each be less than 99.

The Fahrenheit – Celsius conversion is :

$$^{\circ}F = {}^{\circ}C \times 9/5 + 32.$$

Degrees are multiplied by $\pi/180$ to yield radians.

Grads are multiplied by $\pi/200$ to produce radians.

Inches are multiplied by 25.4 to get millimetres.

U.S. gallons are multiplied by 3.785411784 to get litres.

Avoirdupois pounds are multiplied by 0.45359237 to yield kilograms.

Example : 212°F = 100°C

Press	Display/Comments
2nd CA	0
212 2nd F-C	100.
INV 2nd F-C	212.

You can use these conversions to convert square units of one system to square units of another system.

Example . 1520 square inches = 980643.2 square millimetres.

Press	Display/Comments
1520 2nd [illegible] 2nd [illegible]	980643.2

Going through the conversion process twice effectively multiplies by the conversion factor twice. Cubic conversion would work the same way, except that three conversion sequences are required

POLAR TO RECTANGULAR SYSTEM CONVERSIONS

2nd P→R **Polar/Rectangular** - Converts polar co-ordinates to rectangular co ordinates.

$x \rightleftarrows y$ **x Exchange y Key** - Enters and retrieves data for the special calculations. Also used for arithmetic operations and exchanging x and y in root and power calculations.

Polar to Rectangular Key Sequence

R $x \rightleftarrows y$ θ 2nd P→R yields y $x \rightleftarrows y$ x

Rectangular to Polar Key Sequence

x $x \rightleftarrows y$ y INV 2nd P→R yields θ $x \rightleftarrows y$ R

The θ calculated from the rectangular to polar sequence will be

$$\begin{matrix} -90^\circ \\ -\pi/2 \text{ rad} \\ -100 \text{ grad} \end{matrix} \Bigg\} \leqslant \theta < \Bigg\{ \begin{matrix} 270^\circ \\ 3\pi/2 \text{ rad} \\ 300 \text{ grad} \end{matrix}$$

This conversion routine monitors the angular mode of the calculator to determine the angular units desired for both entry and retrieval of data.

Note that arithmetic operations should not be pending when using the polar/rectangular conversion.

Press		Display/Comments
2nd Deg	0	Select degree mode
5 $x \leftrightarrow y$	0.	Enter R
30 2nd P→R	2.5	Enter Θ, display y
$x \leftrightarrow y$	4.330127	Display x
2nd Rad	4.330127	Radian mode
$x \leftrightarrow y$	2.5	Enter x
INV 2nd P→R	0.5235988	Display Θ
$x \leftrightarrow y$	5.	Display R

MEAN, VARIANCE, STANDARD DEVIATIDN

Σ+ **Sum Plus Key** - Enters data points, y_i, for calculation of mean, variance and standard deviation and for the linear regression routines.

2nd Σ− **Sum Minus Key** - Removes unwanted data entries for mean, variance standard deviation and linear regression calculations.

2nd MEAN **Mean Key** - Calculates the mean of the y array.

of data Mean = $\bar{y} = \frac{\sum_{I=1}^{N} y_i}{N}$, I = 1, 2, 3 . . N

2nd VAR **Variance Key** - Calculates the variance of the y array of data using N weighting.

$$\text{Variance} = \frac{\Sigma y_i^2}{N} - \frac{(\Sigma y_i)^2}{N^2}$$

2nd SDEV **Standard Deviation Key** - Calculates the standard deviation of the y array of data using N−1 weighting

$$\text{Standard Deviation} = \sqrt{\text{Var} \times \frac{N}{N-1}}$$

All calculating here must begin and end by pressing 2nd CA to totally clear the calculator. There are 4 pending operations available between entries and calculations of statistics, linear regression and trend line. However, arithmetic operations cannot be pending while actually entering data points. When doing trend-line problems, the implied x value must be reentered with the $x \leftrightarrow y$ key if arithmetic calculations are performed prior to entry of all data points. Statistical values are stored in memories 1 through 7, so external values cannot be stored here without destroying the statistical data.

Data points are entered by pressing [Σ+] after each y, entry and removed by pressing [2nd] [Σ−] after reentry of an incorrect point. The entry number N is displayed after each entry. N = 0, 1, 2 . .

Once entered, the data can be used to calculate the mean, variance and standard deviation by simply pressing the necessary keys.

Example : Analyse the following test scores : 96, 81, 87, 70, 93, 77.

Press		Display/Comments	
[2nd]	[CA]	0	Clear
96	[Σ+]	1.	1st Entry
81	[Σ+]	2.	2nd Entry
97	[Σ+]	3.	3rd Entry (incorrect)
97	[2nd] [Σ−]	2.	Remove 3rd Entry
87	[Σ+]	3.	Correct 3rd Entry
70	[Σ+]	4.	4th Entry
93	[Σ+]	5.	5th Entry
77	[Σ+]	6.	6th Entry
[2nd]	[DEV]	9.8792712	Standard Deviation
[2nd]	[MEAN]	84.	Mean
[2nd]	[VAR]	81.333333	Variance
[RCL]	5	504.	Total of Scores

Note that the standard deviation can be calculated first even though the mean is used to determine the standard deviation.

The data are accumulated in the memory registers with $\Sigma_{y|}$ in 5, $\Sigma_{y|}{}^2$ in 8 and N in 7

The values stored in the memory registers can be recalled and used in other calculator operations

For your convenience, the option has been provided to select N or N−1 weighting for standard deviation and variance calculations. N weighting results in a maximum likelihood estimator that is generally used to describe populations, while the N−1 is an unbiased estimator customarily used for sampled data

Standard deviation and variance can be obtained with N or N−1 weighting. The variance key uses N weighting and the standard deviation key uses N−1 weighting Variance is the square of the standard deviation. So, variance with N−1 weighting is obtained by pressing [2nd] [DEV] [x^2] and standard deviation with N weighting results from [2nd] [VAR] [$\sqrt{x}$]

LINEAR REGRESSION

[x⇄y] **x Exchange y Key** - Enters the x values for linear regression calculations. Also used in conversions, roots and powers and certain arithmetic operations.

[Σ+] **Sum Plus Key** - Enters the y values for linear regression calculations.

[2nd] [Σ−] **Sum Minus Key** - Removes undesired data entries

[2nd] [SLOPE] **Slope Key** - Calculates the slope of the calculated linear regression curve. If the line is vertical, the display will flash because the slope is infinite.

[2nd] [INTCP] **Intercept Key** - Calculates the y-intercept of the calculated linear regression curve. If the line is vertical, the display will flash because there is no y-intercept.

[2nd] [x'] **Compute x Key** - Calculates a linear estimate of x corresponding to a y entry from the keyboard.

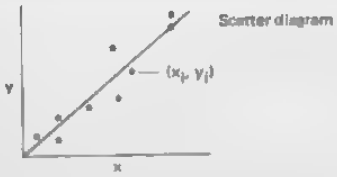
[2nd] [y'] **Compute y Key** - Calculates a linear estimate of y corresponding to an x entry from the keyboard.

[2nd] [CORR] **Correlation Key** - Calculates the correlation coefficient of the data entered in the linear regression routine. The value will be between ± 1 with ± 1 being a perfect correlation.

[2nd] [MEAN] , [2nd] [VAR] , [2nd] [SDEV] - Calculates the mean, variance and standard deviation of the y-array of data.

[INV] [2nd] [MEAN] , [INV] [2nd] [VAR] , [INV] [2nd] [SDEV] Calculates the mean, variance and standard deviation of the x-array of data

In many disciplines it is desirable to express one variable in terms of another even though the variables are independent and are not necessarily analytical functions of each other. An accepted practice is to perform a least-squares linear regression which is designed to minimise the sum of the squares of the deviations of the actual data points from the straight line of best fit. In practice, we are essentially constructing a plot of the variables (called a scatter diagram) and drawing the best straight line which uniformely divides the data points as shown below. Because the data may not be best represented by a straight-line curve, it is desirable to measure how well the linear curve actually does fit the data. This measure is called the correlation coefficient and may be calculated from the independent variables and the linear equation parameters.



Your calculator automatically computes the slope and y-intercept with its linear regression routine. The result is a linear equation of the form

$$y = mx + b$$

It can be shown that the slope and y-intercept are determined as follows.

$$m = \frac{\frac{\Sigma x_i \, \Sigma y_i}{N} - \Sigma x_i y_i}{\frac{(\Sigma x_i)^2}{N} - \Sigma x_i^2}$$

$$b = \bar{y} - m\bar{x}$$

$$\bar{x} = \text{average x value} = \frac{\sum_{i=1}^{N} x_i}{N}$$

$$\bar{y} = \text{average y value} = \frac{\sum_{i=1}^{N} y_i}{N}$$

σ_x^2 = variance of the x values.

$$= \frac{\sum_{i=1}^{N} x_i^2}{N} - \bar{x}^2$$

After the linear regression curve is determined, you can measure the degree of association between the random variables $(x_1, y_1) \ldots\ldots (x_n, y_n)$. This correlation coefficient is usually denoted by r and is calculated using the following expression.

$$r = \frac{m\sigma_x}{\sigma_y}$$

where .

σ_y^2 = variance of the y values

$$= \frac{\sum_{i=1}^{N} y_i^2}{N} - \bar{y}^2$$

From these equations, it is easy to see that the sum of the squares of the data points must not exceed the upper limit of the calculator $\pm 9.9999999 \times 10^{99}$.

The array of x_i, y_i datapoints is entered by pressing

x_i [$x \leftrightarrow y$] y_i [$\Sigma+$]

for each data point. Undesired data points can be removed by reentering the faulty pair, but press [2nd] [$\Sigma-$] instead of [$\Sigma+$], just as in mean, standard deviation and variance calculations.

The x-array data are accumulated with Σx_i in memory 2, Σx_i^2 in memory 3 and Σxy in memory 4. Locations of y-array data are given on page 35.

Example. A quantity of tubing has been ordered cut into 100 cm long sections to be checked for length accuracy and uniformity that should be 6.0 gm/cm ± 0.01. The test requires that 6 samples be analysed at a time.

Sample	1	2	3	4	5	6
Length (cm)	101.3	103.7	98.6	99.9	97.2	100.1
Weight (gm)	609	626	586	594	579	605

What is the average weight of the samples taken? How accurate is the cutting machine? What is the uniformity of the samples? How close were the samples to the standard?

Press		Display/Comments	
[2nd]	[CA]	0	Clear all
101.3	[$x \leftrightarrow y$]	0.	Enter x_1
609	[$\Sigma+$]	1.	Enter y_1
103.7	[$x \leftrightarrow y$]	102.3	Enter x_2
626	[$\Sigma+$]	2.	Enter y_2
98.6	[$x \leftrightarrow y$]	104.7	
586	[$\Sigma+$]	3.	
99.9	[$x \leftrightarrow y$]	99.6	
594	[$\Sigma+$]	4.	
97.2	[$x \leftrightarrow y$]	100.6	
579	[$\Sigma+$]	5.	
100.1	[$x \leftrightarrow y$]	98.2	Enter x_6
605	[$\Sigma+$]	6.	Enter y_6

(continued)

(continued)

Press	Display/Comments	
2nd MEAN	599.93333	Average of y array
+ INV 2nd MEAN	100.13333	Average of x array
=	5.9903462	Average uniformity
2nd CORR	0.9815054	Correlation coefficient

The average weight of the sample is about 599.8 grams. The machine is cutting the length to about 100.1 centimetres. The uniformity is better than 5.99 grams/centimetre, easily within the acceptable tolerance. The correlation coefficient, being very near 1 (perfect correlation) shows that the individual samples were quite close to the uniformity standard.

TREND-LINE ANALYSIS

This process is a variation of linear regression. Calculations must begin and end with 2nd CA . Here, the x values are automatically incremented by 1 for each data point. The calculator normally assigns an x value of 0 to the first y data point. The data points are then entered by pressing $\Sigma+$. The initial x value can be set to any number other than 0 by entering the first value as in normal linear regression x_1 $x\leftrightarrow y$ y_1 $\Sigma+$, then y_2 $\Sigma+$ y_3 $\Sigma+$, etc. The x values are still internally incremented by 1 for each y value. There is no limit on the number of data points that can be entered.

Undesired data points can be removed by the following sequence :

y_{bad} $\Sigma+$, then $x\leftrightarrow y$ -1 = $x\leftrightarrow y$ y_{bad} 2nd $\Sigma-$ y_{good} $\Sigma+$, continue

Example : A company began in 1972. Profits each year since then have been −1.2, −0.3, 2.1, 1.8 and 2.7 million pounds. What profit can be expected in 1977 and in 1980 ? When should profits reach 10 million pounds ?

Press	Display/Comments	
2nd CA	0	Clear All
1972 $x\leftrightarrow y$	0.	Starting x value
1.2 +− $\Sigma+$	1.	y_1
.3 +− $\Sigma+$	2.	y_2
2.1 $\Sigma+$	3.	y_3
1.8 $\Sigma+$	4.	y_4
3.7 $\Sigma+$	5.	y_5
$x\leftrightarrow y$ −	1977.	
1 =	1976.	Faulty entry year
$x\leftrightarrow y$	1979.	

(continued)

(continued)

Press		Display/Comments	
3 7	2nd Σ−	4.	Faulty value removed
2.7	Σ+	5.	Correct value
1977	2nd ŷ'	3.09	Expected profit in 1977
1980	2nd ŷ'	6.96	Expected profit in 1980
10	2nd x̂'	1983.070707	10 million profit year

In the previous sections, the capabilities and operations of your calculator have been explained. This section demonstrates some of the math situations in which your calculator can prove invaluable. For simplicity, M1, M2, M3 will represent memories 1,2 and 3.

VECTOR ADDITION

Add the following vectors :

$$5 \angle 30^\circ + 10 \angle 45^\circ = r' \angle \Theta'$$

Our solution is to first find the individual x and y components of each vector using the polar rectangular conversion routine. Next we sum both x and y components separately to achieve the resultant X and Y values. The equations used are :

$$X = 5 \cos 30^\circ + 10 \cos 45^\circ$$
$$Y = 5 \sin 30^\circ + 10 \sin 45^\circ$$

Finally, we perform a rectangular to polar transformation on the X and Y resultant values to arrive at r' and Θ'. The equations used are :

$$r' = \sqrt{X^2 + Y^2} = 14.865986$$

$$\Theta' = \tan^{-1}\frac{Y}{X} = 40.012765$$

The calculator solution is :

Press	Display/Comments	
2nd CA	0	
5 x⇄y	0.	Enter radius of first vector
30 2nd P→R STO 1	2.5	Enter angle of first vector, complete polar
x⇄y STO 2	4.330127	rectangular conversion. Y stored in M1 and X stored in M2.
10 x⇄y	2.5	Enter radius of second vector
45 2nd P→R SUM 1	7.0710678	Enter angle of second vector complete polar/rectangular
x⇄y SUM 2	7.0710678	conversion. Sum Y components in M1 and X components in M2.
RCL 2 x⇄y RCL 1	9.5710678	Resultant X and Y components recalled for rectangular/polar conversion.
INV 2nd P→R	40.012765	Angle Θ' in degrees
x⇄y	14.865986	Magnitude r'

RECTANGULAR/SPHERICAL COORDINATE CONVERSIONS

To convert (5, 8, 10) from rectangular to spherical coordinates use the following reference system.

Where :

$$r = \sqrt{x^2 + y^2 + z^2}$$

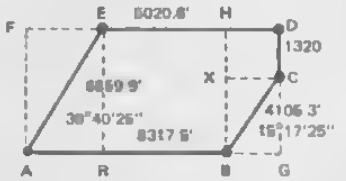
$$\phi = \tan^{-1} \frac{y}{x}$$

$$\theta = \tan^{-1} \frac{\sqrt{x^2 + y^2}}{z}$$

To solve on the calculator .

Press	Display/Comments	
2nd CA	0	
5 x⇆y	0.	Enter x
8 INV 2nd P→R	57.994617	Enter y; value of ϕ displayed in degrees
10 x⇆y INV 2nd P→R	43.33172	Enter z; value of θ displayed in degrees
x⇆y	13.747727	Value of r

AREA OF IRREGULAR POLYGONS



An investor wishes to purchase the tract of land shown for future development With land prices at £0.012 per square foot how much can he expect to spend ? The parts of the figure have been labeled to help you follow the solution.

(Total area) x (Price/unit area) = Total Cost

Total area = AGDF – AEF – BGC

Here is the calculator procedure :

Press	Display/Comments	
2nd CA	0	
6859 9 $x\leftrightarrow y$	0.	
39 4025 2nd $\rightarrow$ 2nd P→R	4379.462	FE
STO 1	4379.462	FE in M1
$x\leftrightarrow y$ STO 2 X	5280.0218	FA in M2
RCL 1 +	4379.462	
5020.6	9400.062	FD
−	49632478.	Area AGDF
RCL 1 X RCL 2 +	23123601.	FE x FA
2 = STO 3	38070677.	AGDF-AFE
4105.3 $x\leftrightarrow y$	38070677	
15 1725 2nd $\rightarrow$ 2nd P→R	1082.6061	BG
STO 1	1082.6061	BG in M1
$x\leftrightarrow y$ X RCL 1 ÷	4287099.9	BG x CG
2 =	2143550.	Area BGC
+/− + RCL 3 =	35927127.	AREA
X		
.012 =	431125.53	Cost of plot

At this point you've looked at a lot of keys on your calculator — and you've explored how to use them in a variety of problem solving situations. Now we're ready to take a look at how you can expand the use of these features even more — with programming.

When many people first hear the word "programming" they conjure up visions of large machines, punched cards, complex procedures, etc. However, programming your calculator is expressly designed to be a natural, straightforward process — that can save you considerable time whenever you handle a repetitive calculating situation. Basically, your calculator just *learns* or remembers keystrokes you put into it. It will then execute these keystrokes for you again at any time — as many times as you require — with the touch of a single key. The calculator is actually "pushing its own buttons" for you.

Let's see how this procedure works by jumping right into a simple example

Let's suppose that you are shopping and spot a store that is having a "40 percent off" sale. To calculate the sale price of any item, you'd simply enter the price into the display and press [−] 40 [%] [=] . A coat which normally sells for £56 would cost : 56 [−] 40 [%] [=] 33.6 or £33.60. As you check various items in the store, you press the same five keys again and again, [−] 40 [%] [=] . If you wander through the store and calculate 16 different discount prices, you'll use these same keystrokes 16 times. However, with your programmable calculator, you can "program" (or more simply, teach) the calculator to remember the [−] 40 [%] [=] , keystroke sequence for you. From then on in, all you'll need to do is enter the price, tell the calculator to begin, and the calculator will push the "programmed" buttons automatically for you.

A program for your calculator, then, is just a list of the series of keystrokes in the order needed to perform a particular calculation Once you know these keystrokes you can program your calculator to remember them. To do this you simply press the [2nd] [LRN] (Learn) key sequence. When you do this, you "turn on" a special memory in your machine that remembers the keystrokes that follow. You're telling the calculator "please remember the keystroke instructions I enter next".

At this point you just enter the keystrokes you'd need to solve your problem. When your program keystrokes are all entered, you press [2nd] [LRN] again to turn "off" the program memory — and you're ready to use or "run" your stored program.

Let's program our 40 % discount problem and see how we can teach the calculator to remember the [−] 40 [%] [=] keystrokes. To do this, follow these steps

Press		Display/Comments
2nd CA 2nd FIX	2	0.00 Clears all calculator memory registers and fixes the decimal point at two places
2nd LRN		00 00 Tells the calculator to "remember" all of the following keystrokes. The special display format (00 00) confirms that the calculator is in "learn mode". (This is discussed more in a moment).
– 40 % | =		06 00 Your calculator "counts steps" as you enter them, at this point step 5 is "up next".

After your calculator completes these keystrokes, you want it to stop and show you the result. You tell it to do this by finishing your program with a *stop* instruction 2nd R/S .

2nd R/S		06 00 Tells calculator to stop
2nd LRN		0 Leave learn mode. The program is complete. This second use of the learn sequence "turns off" the program memory to leave the "learn mode"

Now, to use or "run" your program, follow these steps

When we left the learn mode, after keying in the program, the step counter (program pointer) was sitting waiting for the next step, step 06. The calculator has "learned" our program step-by-step and these program steps we've stored in steps 00, 01, 02, 03, 04, and 05. Before we can run the program we must get back to step 00 so that the program pointer will run (or execute) the program from the beginning. You can do this by pressing the reset key, 2nd RST . Then the program is ready to run

Press		Display/Comments
2nd RST		0.00
53.95		53.95 Enter the list price of the merchandise. Now press Run/Stop to run the program. (– 40 % =)
2nd R/S		32.37 Sale price.

As you go through the store, you can quickly and easily calculate the sale price of any item by pressing 2nd SST, entering the list price, then pressing 2nd R/S.

To prove to yourself just how easy the program is to use, find the price after discount on these items. £155.97, £86.49, and £13.88

Press				Display/Comments
2nd RST	155.97	2nd R/S		93.58 Discount price
2nd RST	86.49	2nd R/S		51.89 Discount price
2nd RST	13.88	2nd R/S		8.33 Discount price

To help you to better understand what's going on within the calculator when you run a program, let's take a closer look at the four special programming keys on your calculator. They're shown here as they appear on the keyboard and include R/S, RST, LRN and one we haven't used yet SST.

Programming Keys

PROGRAMMING KEYS

2nd LRN **The Learn Mode Key** - Pressing the sequence 2nd LRN one time puts the calculator in what we'll call the "learn" mode of operation. This allows you to begin writing a program into program memory which is "learned" and remembered by the machine and can be run later. Pressing the sequence 2nd LRN again takes the calculator out of the learn mode. (The display is cleared to a single zero when you leave the learn mode).

When you press 2nd LRN the first time and enter the learn mode the display changes to a unique format :

The two digits on the left tell you the program step number you are working on. As you are programming the machine these two digits will always indicate the number of the next available program step. Thirty-two program steps (numbered 00 through 31) are available for your use.

The right two digits in the display will be zeros as you program the machine, but as you'll be seeing in a moment, these two digits will tell you which keystroke is at each program step when you review your program with the 2nd SST key.

The keystrokes will be indicated by a two-digit number code (called the key code) representing the row and column number of the key. (More on this later).

2nd R/S **The Run/Stop Key** - When your calculator is out of learn mode, the 2nd R/S key is the start/stop switch for any program you may have in the machine. If the program is stopped, pressing 2nd R/S will start it running. If the program is running along, pressing 2nd R/S or CE will stop it. The 2nd R/S key can also be put in a program where you want the calculator to stop to display an answer. The calculator will run through your program steps until it comes to a 2nd R/S instruction, at which point it will stop.

2nd RST **The Reset Key** - In order for you and your calculator to be able to keep track of your programming steps, they are numbered sequentially from 00 to 31. As you key in a program (and as a program is running) the program step counter, or program pointer, advances step-by-step from 00 to 31 (or to a R/S instruction before step 31). The 2nd RST key instructs the calculator to reset the program counter to step 00. Pressing 2nd RST, then, takes you back to the beginning of your program.

2nd SST **The Single-Step Key** - If you press this key sequence while your calculator is in "learn" mode, you "step through" your program one step at a time. This allows you to check on the keystrokes in any program you've entered, as we'll discuss below. When you press 2nd SST out of "learn" mode, you step through and execute your program one step at a time.

To see the 2nd SST key sequence in action, let's go back and key in the simple program we used previously to compute price markdowns.

Press	Display/Comments
2nd CA 2nd FIX 2	0.00 Clears machine, fixes decimal at 2 places.
2nd LRN	00 00 Enter learn mode
− 40 % L= 2nd R/S	06 00 Key in program steps
2nd LRN	0. Exit learn mode

Now to go back and review this program using the 2nd SST key sequence, just perform the following keystrokes.

Press	Display/Comments
	Reset and enter learn
2nd RST 2nd LRN	00 65 Step 00
2nd SST	01 04 Step 01
2nd SST	02 00 Step 02
2nd SST	03 22 Step 03
2nd SST	04 85 Step 04
2nd SST	05 86 Step 05
2nd SST	06 00 Step 06
2nd SST	07 00 Unprogrammed steps

(Leave your calculator on for the next example)

Notice again that four digits appear in the display. The left two digits tell you the calculator's program step number location. The right two digits are a number code that tells you what keystroke that particular step will execute when the program runs.

If you continue to press the 2nd SST key sequence while your calculator is in learn mode, it will go to step 31, then repeat back to step 00.

Refer to page 57 for special programming notes

KEY CODES

The key code your calculator uses to indicate each step is a fairly straightforward one. The two digits simply represent the row and column numbers of the key in question (except for the number keys [0] through [9] which are represented by their number e.g. 06 represents [5] etc.). For second functions on your calculator, the keycodes for the column are 6, 7, 8, 9, and 0 rather than 1 through 5, as shown in the diagram below.

Code for 2nd LRN is 19

31 (row 3 column 1 is [x≥y])

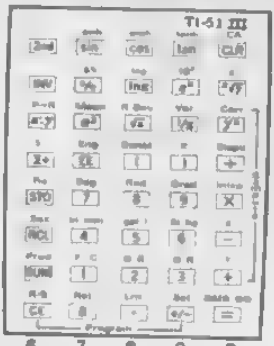
42 (row 4 column 2 is [EE])

06 (number key [5])

65 (row 6 column 5 is [−])

03 (number key [3])

85 (row 8 column 5 is [=])



Column numbers for second functions 6 7 8 9 0

Columns 1 2 3 4 5

See Appendix D for a complete list of key codes

The display "00 65" tells you that step 00 is [−], display "01 03" tells you that step 01 is 3, and so forth. All of the keys used in your program are displayed with their key codes when you single step through "learn mode". You can check to see if your program is entered properly using this method.

If a step is not entered correctly (or you want to change it) you can enter a new keystroke at any step by simply keying it in. A new keystroke will "write over" and replace any step that's already there. (The display will then move on to the next step.)

NOTE . When entering the second function keys, pressing [2nd] and then the desired second function uses only one of your 32 allowable program steps

Let's go back and modify the program you now have in the calculator to discount 30 % instead of 40 % (change the 4 to a 3). (Notice : At this point your calculator may have switched over to its power saving display -- pressing [2nd] [2nd] restores the display -- even in the learn mode)

Press	Display/Comments
[2nd] [LRN]	0. Leaves learn mode
[2nd] [RST]	0.00 Return to step 00
[2nd] [LRN]	00 65 Enters learn mode

Now we'll single step to the 4 and change it to a 3.

[2nd] [SST]	01 04 This is the step we want to change to a 3.
[3]	02 00 The 3 has replaced the 4 in step 01 and the calculator has moved on -- showing the contents of step 02.
[2nd] [LRN]	0. Leaves learn mode

Now the program discounts 30 % instead of 40 %. To see this, let's use our modified program to calculate a sale price with 30 % discount. For example, find the sale price with 30 % discount. For example, find the sale price of an item regularly costing £25.95.

Press	Display/Comments
[2nd] [RST]	0.00 Resets to step 00
25.95	25.95 Enters the regular price
Now start the program	
[2nd] [R/S]	18.17 The sale price is £18.17

To continue to find other 30 % discounted items, press 2nd RST , enter the price, and press 2nd R/S . However, if you have several discount items, you can save yourself some effort by including 2nd RST as the last step of the program

USING THE RESET KEY – RST – INSIDE A PROGRAM

When RST is entered as a program step, it tells the calculator to return to step 00 By placing a 2nd RST instruction right in your programs, you can eliminate the need for pressing 2nd RST each time you use the program

Let's write a program to discount the number you enter in the display by 25 %. This time to use the program, you'll enter the regular price and press R/S . You want the calculator to then compute the discounted price and stop, and have it ready to reset automatically for the next calculation. Here's how you can do it.

ENTERING YOUR PROGRAM

Press		Display/Comments
2nd CA 2nd FIX	2	0.00 Clears all registers and fixes decimal at two places.
2nd LRN % =		00 00 Enters learn mode.
– 25		05 00 Enters program you want.
2nd R/S		06 00 Tells calculator to stop.

After running the program once, the program counter would stop at this R/S instruction. Now you can tell the calculator to reset "automatically" to the beginning so you can enter a new price, start the program again, and have the program reset automatically and compute the discount price.

2nd RST		07 00
2nd LRN 2nd RST		0.00 Exit learn mode and reset for the first calculation.

Running Your Program : To use this program to find the sale price of items costing £25.95, £15.42, and £17.87, just enter the regular price of each item and start the program.

Press	Display/Comments
25.95 2nd R/S	19.46 Sale price
15.42 2nd R/S	11.57 Sale price
17.87 2nd R/S	13.40 Sale price

Automatic reset is a convenient tool to help make it easy for you to use this program for as many items as you need to compute the discount

The examples thus far give you a basic understanding of your calculator's four programming keys. However, before moving on to more programming examples, let's briefly look at how you enter the numbers you need for your program calculations

DATA ENTRY

Every program you write of necessity involves using some data for calculations. Because of this you need to be aware of how to enter data for your program to use. Basically there are two ways to enter data into a program : either from the display, or by recalling the data from memories.

One of the simplest methods of entering data for your program is to just use the number in the display. This works well even if you need to enter more than one number since you can always include a [2nd] [R/S] in the program to stop and allow the entry of the second value.

Another way to enter data is to store it in memories (either as part of the program or before you start the program) and then let the program recall the numbers from memory as needed for the calculation.

With this in mind, let's go on to more program examples

Mail Order Program

You work in a mail-order discount house and fill 75 to 100 orders per day, discounting the list price by 20 % and adding £1.50 for shipping and handling. An average calculation looks like this .

56.15 [−] 20 [%] [+] 1.5 [=] 46.42.

You push the same keys over and over all day long. Why not let the *calculator* push the keys for you ? You can by using this simple calculator program.

Press		Display/Comments
[2nd] [CA] [2nd] [Fix]	2	0.00 "Clears All" and fixes decimal at 2 places
[2nd] [LRN]		00 00 Puts calculator in "Learn" mode – allows you to teach it step sequence.
[−] 20 [%]		04 00 Discounts cost by 20 %
[+] 1.5 [=]		09 00 Adds handling charge
[2nd] [R/S]		10 00 Stops Program
[2nd] [RST]		11 00 Resets program back to step 00 (automatic reset)
[2nd] [LRN]		0. Leave learn mode

Now use the program to find the final order price for these orders, £29.95, £32.50, £167.95 and £20.00

Press	Display/Comments
2nd RST	0.00 Resets calculator to first program step for first calculation
29.95 2nd R/S	25.46 DISCOUNT PRICE
32.50 2nd R/S	27.50 DISCOUNT PRICE
167.95 2nd R/S	136.86 DISCOUNT PRICE
20.00 2nd R/S	17.50 DISCOUNT PRICE

SPECIAL TRICKS

Pause Function

The pause key found on some calculators allows the user to look at a number within a program for a moment before the program moves on. You may accomplish the same effect with your calculator by entering consecutive [=] 's following a number you want to observe during program operation.

Example · You want to double a sum of money each day for several days, and watch it grow during the process

Press	Display/Comments
2nd CA	0.
2nd LRN	00 00 Enter learn mode
X 2 =	03 00 Doubles displayed value
=	04 00
=	05 00
=	06 00
=	07 00 Holds new value on display
=	08 00 (dim. but readable)
=	09 00
=	10 00
=	11 00
2nd RST	12 00 Resets calculator to 00
2nd LRN	0. Exit learn mode

To run the program, enter the initial number to be doubled, press 2nd RST 2nd R/S and watch the display. You can actually see the number grow !
If you want to double the number 20 times, you must count the number of times different numbers appear on the display

Continuous Loops

The previous program is a good example of what is called a "continuous loop". A loop is a program that will continue to repeat over and over without extra instructions. In this way, your calculator can perform many calculations in the time it would take you to manually make one or two "loops" through the problem.

However, as in the preceding program, it's difficult to count (and remember) one number while watching another number flash on the display. Using some clever maneuvering, though, we can program the calculator to keep count for us as it works the problem.

Here's the above example re-written slightly to include a "counting" sequence.

Press	Display/Comments
2nd CA	0.
2nd LRN	00 00 Enter learn mode
× 2	02 00 Double number in display
= = = =	
= = = =	10 00 "Pause"
STO 1	12 00 Store number in display
RCL 2	14 00 Recall step number stored in memory 2
+ 1	16 00 Add 1 to number of steps
= = = =	
= = = =	24 00 "Pause"
STO 2	26 00 Store new step number
RCL 1	28 00 Recall number to be doubled
2nd RST	29 00 Reset to step 00
2nd LRN	0. Exit learn mode

To run the program .

Press	Display/Comments
2nd RST	0. Reset calculator to step 00
CLR STO 1 STO 2	0. Clears memories 1 and 2
1	1. Number to be doubled
2nd R/S	"2" 1 doubled (Value flickers in display)
	"1" Loop 1
	"4" 2 doubled
	"2" Loop 2
	"8" 4 doubled
	"3" Loop 3
	"16" 8 doubled

The advantage of this program is that it counts for you. You must still watch the display carefully and stop the program on the loop *before* the desired loop. You may then press [2nd] [SST], stepping the program through to the "doubled" number. This program is easier to use than the first. However, the ideal way to run this program would be to have the calculator stop after it has completed a specified number of loops.

FINITE LOOPS

Your calculator does not have special keys which would allow you to set certain conditions like stopping after a specified number of loops. You do, however, have access to certain operations which will stop the program whenever these operations are encountered. These are called "illegal" or undefined operations. Here is a short list of undefined operations:

Press	Display/Comments
0 [1/x]	"9.9999999 99" (Flashing)
0 [ln x]	"– 9.9999999 99"
90 [tan]	"9.9999999 99"

For a list of "undefined operations," see Appendix B.

By using these "undefined operations" you can cause the program to "stop in its tracks". Here's how:

If your calculator encounters the instruction "90 [tan]", it will stop on that program step and flash 9's in the display because the tangent of 90° is an undefined operation. In the last program, we were doubling a number a certain number of times. If we wanted to see what 1 doubled 20 times was, we had to manually stop the calculator on Loop 20.

You can have the machine stop itself after 20 steps by inserting an illegal operation at that point. We know that 90 [tan] will stop the machine, right? OK, to stop the machine after 20 steps, just subtract 20 from 90. Insert 70 into a memory and have the machine add 1 to it each time a loop is completed, then find [tan] of that number. When it tries to compute tan 90°, the program will stop. Here's how the program would look:

Press	Display/Comments
2nd CA	00 00 Enter learn mode
2nd LRN	
× 2 =	03 00 Double number in display
STO 1	05 00 Store results of doubling operation so it can be recalled after program stops.
RCL 2	07 00 90 minus number of loops desired stored here
+ 1 =	10 00 Increment memory 2 by 1
STO 2	12 00 Store
tan	13 00 Check to see if memory 2 is equal to 90. If it is, program will stop on this step. If not the program continues
RCL 1	15 00 Recall number to be doubled
2nd RST	16 00 Reset machine to step 00
2nd LRN	0. Exit learn mode
2nd RST	0.

To run it

Press	Display/Comments
70 STO 2	70. Store 90 minus number of loops desired in memory 2
2nd RST	70. Reset to step 00.
1	1. Enter initial number to be doubled
2nd R/S	"999999999 99" Run program..... ....program stops
CLR RCL 1	1048576. 1 doubled 20 times

The above results can also be achieved by reducing a number in a memory to zero, and taking its reciprocal each time (1/0 = "9 9999999 99").

Zero Check

The method previously shown can be used if you know the exact number of loops you want to execute. If you do *not* know how many loops you want to make, and simply want the answer to a problem, you can use what we'll call a "Zero Check"

Certain types of problems can be solved by using an iterative approach. The equation you use must be structured so that it "closes in" on the answer in succeeding steps. As succeeding steps become closer and closer to the answer, the difference between the steps approaches zero.

When zero difference is achieved, the calculator has solved the problem. By using our "Zero Check" the display can be made to flash when the calculator has solved the problem.

Consider the following equation .

$$f(x) = x^3 + x - 1 = 0$$

By applying Descartes' rule of signs we find that this equation has only one real positive root. We can approximate the real root by writing the equation as :

$$x = \frac{1}{1 + x^2}$$

We can now construct an approximation routine using the form

$$X_{n+1} = \frac{1}{1 + X^2_n}$$

The program would look like this .

Press	Display/Comments	
2nd CA 2nd FIX 4	0.0000	Clear all,fix decimal at 4 places.
2nd LRN	00 00	Enter learn mode
x² + 1 = 1/x	05 00	Equation
E E	05 00	Sets accuracy to 4 places
STO 1	06 00	Store first approximation in memory 1
− RCL 2	11 00	Subtract second approximation from first approximation
= 1/x	13 00	"Zero Check" – if memories 1 and 2 are equal
		Difference is zero-display flashes.
RCL 1 STO 2	17 00	Store latest approximation in memory 2.
2nd R/S	19 00	Reset machine to step 00
2nd LRN	0.	Exit learn mode

To run it :

Press	Display/Comments
2nd R/S	0.0000 Reset program to 00
2nd R/S	"9.9999 99" Run program . . . program stops
CLR RCL 1	0.6823 Answer correct to 4 places (to see 5th digit, press 2nd FIX 5)

PROGRAMMING NOTES

1. When programming, you may use memory registers 0 through 7. Registers 8 and 9 are reserved for program steps 25-31 and 17-24 respectively. If registers 8 and 9 are used, the program can be up to 17 steps (00-16) long.
2. When programming a problem that requires second, inverse functions (such as arc sinh) be sure to press the *inverse key before* you press the 2nd function key.
3. When writing a program, you may use the CLR key within the program, but not the 2nd CA key, as the 2nd CA key will clear the whole calculator, including the program.
4. When working Linear Regression problems, Registers 1 through 7 are dedicated to use by the machine. Also, a maximum of 24 program steps (00-23) is allowed when working Linear Regression problems.

PROGRAMMING APPLICATIONS

Approximating Derivatives

Your calculator can also aid in the approximation of derivatives. For example, let's approximate the derivative of $f(x) = \sin x$ at $x_0 = 45^\circ$, or $\pi/4$ radians. Recall that if $f(x) = \sin x$, then $f'(x) = \cos x$. Also,

$$f(x) = \lim_{\Delta x \to 0} \frac{f(x_0 + \Delta x) - f(x_0 - \Delta x)}{2\Delta x}$$

$$\approx \frac{\sin(\pi/4 + .0001) - \sin(\pi/4 - .0001)}{2(.0001)}$$

Enter the following program .

Press	Display/Comments	
2nd CA 2nd LRN	00 00	Enter learn mode
RCL 1 + RCL 2 =	06 00	
sin − (RCL 1	11 00	
− RCL 2)	15 00	
sin = + (	19 00	
2 X RCL 2	23 00	
) =	25 00	
2nd R/S	26 00	Stop
2nd RST	27 00	Reset to step 00
2nd LRN	0.	Exit learn mode

To run it

Press	Display/Comments	
2nd π + 4 = STO 1	0.7853982	Calculate x_0 in radians and store in memory 1.
.0001 STO 2	.0001	Store Δx in memory 2.
2nd Prgm 2nd Rad	0.	Reset machine to 00, select radians mode
2nd R/S	0.7071076	Value of f' (π/4)

To find the difference between f' (π/4) and cos (π/4).

Press	Display/Comments	
− RCL 1	0.7853982	x_0 in radians
cos =	0.0000006	difference

Solving Differential Equations

Suppose that we have a differential equation of the form $y' = f(x,y)$, $y(0) = a$. Approximate solutions can be obtained by using the following recursive equation .

$$Y_{n+1} = Y_n + hf\,(X_n + Y_n)$$

$$Y' = X + Y,\ Y(0) = 0\ .\ h = 2$$

recursion relation becomes

$$Y_{n+1} = Y_n + h\,(X_n + Y_n)$$

Where : $S_n = nh$

By inspection, the value of $Y_{n+1} = 0$, with $n = 0$. Therefore, the calculator solution will begin with $n = 1$ and $h = 0.2$.

Enter the following program :

Press	Display/Comments	
2nd CA	0.	Clear all memories
2nd LRN	00 00	Enter learn mode
RCL 1 + RCL 2	05 00	Yn + h
X ( ) CE	08 00	CE clears all entries back to last *number* entered (h)
X RCL 3	11 00	n
+ RCL 1	14 00	Yn
= STO 1	17 00	new Yn
1 SUM 3	20 00	Add 1 to n
RCL 1	22 00	Yn
2nd R/S	23 00	Stop program
2nd RST	24 00	
2nd LRN	0.	Exit learn mode

To run it

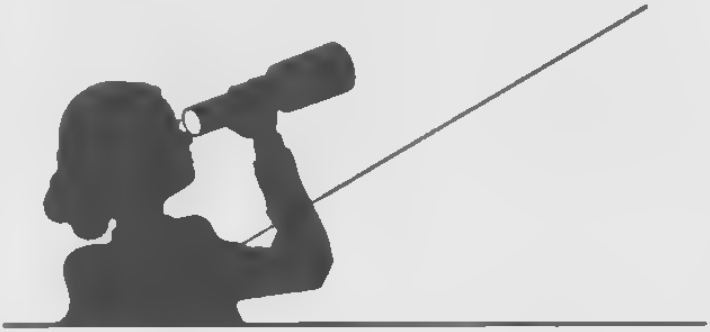
Press	Display/Comments	
.2 STO 2	0.2	Store h in memory 2
1 STO 3	1.	Store n in memory 3
2nd RST 2nd FIX 3	1.000	Reset machine to step 00 Fix decimal at 3 places
2nd R/S	0.040	Yn (n = 0)
2nd R/S	0.128	Yn (n = 1)

(See Tables p. 60)

n	Xn	Yn	$Y_n + h(X_n + Y_n)$	Actual Y-Value
0	0.0	0.000	0.000	0.000
1	0.2	0.000	0.040	0.021
2	0.4	0.040	0.128	0.092
3	0.6	0.128	0.274	0.222
4	0.8	0.274	0.488	0.426
5	1.0	0.488	0.786	0.716
6	1.2	0.786	1.613	1.120
7	1.4	1.183	1.700	1.655
8	1.6	1.700	2.360	2.353
9	1.8	2.360	3.192	3.250
10	2.0	3.192	4.230	4.389

NOTE : The accuracy of the above algorithm can be increased by selecting a smaller value of h.

Measuring & Forecasting Trends



Knowledge about (and some control over) what will happen *in the future* is an important aspect of managing any type of business enterprise *today*. The more you can predict about how prices will vary ; how well a sales force will perform ; how advertising will effect sales ;etc', the easier it will be to make sound decisions in a variety of business situations' Knowing how well one variable will relate to another can allow you to make better things happen in your everyday life, as well as your business !

The following examples illustrate some techniques aimed at *making predictions of future performance based on past "track records"* We'll also discuss tools for making decisions about *whether or not two variables are related*, and if so, how much you can rely on the relationship in "driving" your business Your calculator is equipped with special keys that can make handling the maths involved a cinch ! These keys handle what statisticians would call the techniques of *linear regression* and *correlation*. If you're not really familiar with all the "ins and outs" of what these words mean, that's not important. What is important is that they're the names of very useful mathematical tools that your calculator makes easy to use

Keys to Linear Regression -
or, Straight Line Graphs Made Easy

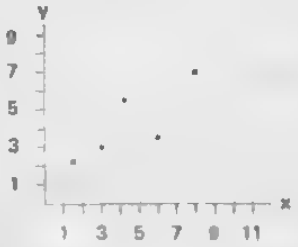
The "linear regression" part of your calculator includes the [Σxy] and [Σ+] keys, as well as all the second function keys on the right side of the machine labeled

CORR LOG STAT x' y'

Basically what these keys do is allow your calculator to mathematically draw the "best fitting line" through a series of data points. You just key in your data with the [Σxy] and [Σ+] keys. While you're doing this, your calculator keeps up with you constantly - "drawing" the best fitting straight line through these points. The *basic* elements of how these special keys are used were discussed for you in *Chapter I*. In this chapter we'll go through a brief review of the process with a little more detail - so you're sure you're using the keys correctly before we move on to *calculating better decisions*. (The techniques described in the chapter will allow you to make predictions on any process or operation that can be assumed to follow a straight line pattern of behaviour).

Example : Let's say you've got some data - it can be about any sort of process or operation, but it's the best "track record" you've got - and you need to make some future predictions based on it. Data such as this is often expressed in terms of pairs of numbers labeled with the letters *x* and *y* such as those tabulated below The points could be "plotted ' pictorially as shown

x	*y*	
1.5	2.25	
3.0	3.0	Five data
4.25	5.5	points you
6.0	3.5	*know*
8.0	7.0	
12	?	Predictions
?	11.25	you *need* to
		make.



Now, *x* and *y* can be any of a variety of variables with some relation between them. (Thousands of pounds of advertising vs sales volume in hundreds of units, employees' scores on an exam vs performance, etc.) Your task is usually to make predictions based on the data you've got. Typical things you might need to know in this case could be :

For a given *x* value (say *x* = 12), what will the value of *y* be ? *or*
For what *x* value will *y* reach some specific number (say 11.25) ?

You might also like to know something about how accurate the predictions are, as well as how you can make additional predictions easily at a later time.

Here's how to use your calculator to help.

Steps in Calculating Predictions and Forecasting Trends

First, enter the information you *have* (your data) as follows :
Enter each *x* value, push [x⇆y], enter the *corresponding y value*, then push [Σ+]
Repeat the process for all the data.
For the data tabulated in our example.

Press	Display/Comments
[2nd] [CA]	0
1.5 [x⇆y] 2.25 [Σ+]	1. Notice that the
3.0 [x⇆y] 3.0 [Σ+]	2. calculator keeps
4.25 [x⇆y] 5.5 [Σ+]	3. track of how
6.0 [x⇆y] 3.5 [Σ+]	4. many data points
8.0 [x⇆y] 7.0 [Σ+]	5. (pairs of *x* and *y* values that you enter)

As you enter the data, your calculator is storing and analysing it.

Now, if you *need to predict a y value, for a given x value* just enter the *x* value, and press [2nd] [ŷ,r]

In our case we need to know : for $x = 12$, what will y be ?

Press	Display/Comments
12 2nd $\hat{y}$	8.8882025 - the y value for $x = 12$.

If you want to go the other way that is, you have a y value and need to know the corresponding x value
enter the y value, then press 2nd x'
In our case we want to know at what x value y will reach 11.25.

Press	Display/Comments
11.25 2nd x'	16.79358 - the x value for $y = 11.25$

To get a picture of *how well the data correlates*
press 2nd CORR This displays the *correlation coefficient* for the line.

In our case :

Press	Display/Comments
2nd CORR	0.8097825

About the Correlation Coefficient

The 2nd LOGR key sequence displays the *correlation coefficient* of the two sets of data (x's and y's). A value close to plus 1 indicates a high positive correlation and a value close to minus 1 indicates a high negative correlation. A value of zero indicates that the two sets of data are not related.

For example : Suppose your company gives two tests to new employees - *Test A* and *Test B*. If there is a high positive correlation between the two tests, then you can predict that an employee who scores high (or low) on *Test A* will also score high (or low) on *Test B*. On the other hand, if there is a high negative correlation between the two tests, you can predict that an employee who scores high (or low) on *Test A* will score low (or high) on *Test B*. If there is no correlation (correlation coefficient equals 0), then you can say nothing about how an employee's performance on *Test A* relates to his or her performance on *Test B*

Slope and Intercept

To find out more about the line, press 2nd SLOPE and 2nd INTCP to display the slope and intercept of the line.

Press	Display/Comments
2nd SLOPE	0.6225775
2nd INTCP	1.4172723

The slope of the line is the ratio of its "rise" to its "run", while the intercept is where it crosses the *y* axis. Any straight line may be expressed as an equation most commonly written in the form .

$y = mx + b$

Where m is the slope value and b is the intercept value.

Using your calculated values you could then write an equation for the line best fitting your data as follows .

$y = (.62)x + 1.42$

(Where we've rounded off the slope and intercept)

You could then use this equation to predict a *y* value for any selected *x* value with a simple calculation later on, without having to re-enter the data each time

Putting It All Together

So, using the linear regression and correlation keys can give you quite a bit of information about (and analysis of) your data. To use the calculator to do this, you just .

Enter each *x* value and press [x,y]

Enter each *y* value and press [Σ+]

The calculator mathematically draws the "best fitting line" for your data-points and you can use the information about this line to predict

Given any *x* value, what is the corresponding *y* value ?

(enter the value of *x*, press [2nd] [y'])

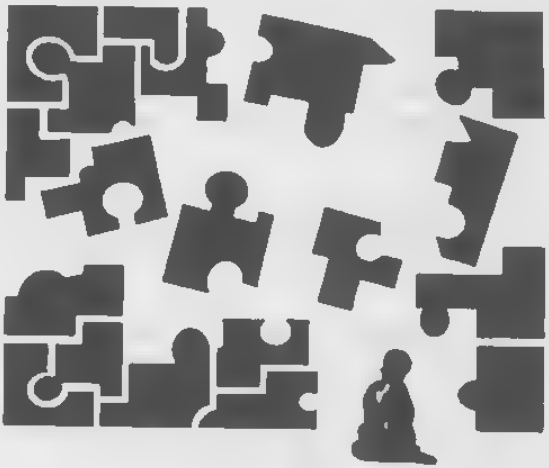
Given any *y* value, what is the corresponding *x* value ?

(enter *y*, press [2nd] [x'])

You can also get an idea of how well the data correlates.

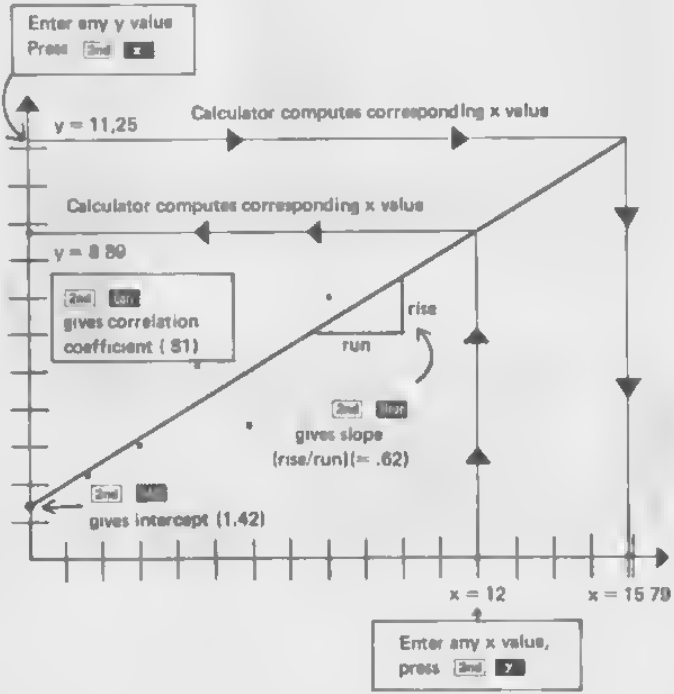
(Press [2nd] [CORR] the closer the display reads to plus or minus 1, the better the correlation).

To calculate the slope and intercept of the line press [2nd] [SLOPE] and [2nd] [INTCP]



The following diagram illustrates all of this for you .

After entering the x, y coordinates of the known values



The rest of this chapter covers a few examples of how these procedures can be helpful in calculating better decisions.

PREDICTING SALES FROM ADVERTISING (Linear Regression)

II-2

Let's say your company has recently started advertising in a new medium (say a series of magazines), on a weekly basis. The marketing manager has a record of the amount spent on advertising each week (x) and the corresponding sales volume (y) and there seems to be a fairly good relationship. His question of you is : what would the expected sales volume be if £4750 is spent on magazine advertising next week ?

Amount Spent on Advertising (x)	Weekly Sales Volume (y)
£ 1000	101,000
£ 1250	116,000
£ 1500	166,000
£ 2000	209,000
£ 2500	264,000
£ 4750	? ? ?

Target : You want to predict what will happen, in unit sales (y), if your advertising budget (x) is increased to £4750 - using the "best straight line" approximation

Tools : You'll be using the *linear regression* feature of your calculator to help predict the result. First, you'll be *entering your data*

Enter £ in advertising :
press [x,y]

Enter corresponding unit sales
press [Σ+]

To make your prediction you'll enter your trial advertising £ value, and to find the predicted result just press [2nd] [y']

You'll note a slight pause when you press the [$\hat{y}$] key - before the result is displayed. That's because your calculator has the chore of handling the linear regression calculation (not you). Here's the formula for what it's doing.

$$\hat{y} \left[\frac{\frac{\sum x_i \sum y_i}{N} - \sum x_i y_i}{\frac{(\sum x_i)^2}{N} - \sum x_i^2} \right] \times (\text{your "x" value}) + \left\{ \frac{\sum y_i}{N} - \left[\frac{\frac{\sum x_i \sum y_i}{N} - \sum x_i y_i}{\frac{(\sum x_i)^2}{N} - \sum x_i^2} \right] \left(\frac{\sum x_i}{N} \right) \right\}$$

(Think of the "fun" you'd have doing this calculation yourself !)

(To find out about how well your data correlates to a straight line, you can press [2nd] [CORR] to display the correlation coefficient. A value near one means a fairly good linear correlation).

Keying It In : First, enter the data

Press		Display/Comments	
[2nd] [CA]		0	Clears entire machine
[2nd] [FIX] 2		0.00	Sets display to read out 2 decimal places.
Enter the 5 data points .			
1000 [$x \leftrightarrow y$] 101,000 [$\Sigma+$]		1.00	Calculator
1250 [$x \leftrightarrow y$] 116,000 [$\Sigma+$]		2.00	displays the
1500 [$x \leftrightarrow y$] 165,000 [$\Sigma+$]		3.00	number of
2000 [$x \leftrightarrow y$] 209,000 [$\Sigma+$]		4.00	(x, y) points
2500 [$x \leftrightarrow y$] 264,000 [$\Sigma+$]		5.00	entered
			Now, to find the y value for x = £4750
4750 [2nd] [$\hat{y}$]	514672.41		

Based on the best straight line approximation, the projected weekly sales volume for £4750 spent on advertising is 514,672.41 units.

Now to check out how good an estimate you and your calculator made :

Press	Display/Comments
[2nd] [CORR]	0.99

Nearly perfect positive correlation !

Decision Time : You're now in a position to make predictions about your future sales based on advertising. Your correlation coefficient seems to indicate that the prediction will be a good one - but remember the total number of data points you're working with is small - you have only 5 points upon which to predict the future. As it turns out, there's a way to further analyse your correlation that allows you to take the number of data points into account (see *Going Further* section).

So, to get down to a decision at this point - you might take a look at the increased cost, weight that against the increase in sales that you predict will result - and see if it's "worth it"

Press	Display/Comments	
CLR	0	
4750 − 2500 =	2250.00	Amount of advertising increase
514672.41 − 264 000 =	250672.41	Increase in unit sales predicted

"Cause and Effect"

Note an important point here. Strictly speaking all we've shown in this example is that a definite *relationship* exists between advertising and sales. Be careful about drawing conclusions about *cause and effect*. In this case, you can probably be pretty sure that your advertising is pushing your sales up - but in other cases, the "cause and effect" relation may not be so obvious. Two variables that are related to a *third* can show a relation to each other - without a "cause and effect" relation between them.

For example, you may have data on children that relates manual dexterity (let's say the time to finish a simple jigsaw puzzle) directly to mathematical ability (performance on a maths test). The relation may show quite a good correlation coefficient. It may turn out, however, that *age* is the dominant factor "driving" the variables. Further analysis may show that the older children naturally display both better manual coordination and mathematical skill - and that if your sample is restructured to include only children of the same age - an entirely different relationship may result. So be careful about how you apply your results in making decisions. Consider the makeup of your sample and exactly what you're measuring and testing.

Going Further : Correlation Factor Validity

As we briefly mentioned - in this example you're predicting the future based on only five data points from the past - and that's not much to go on. In general, the less data you have to go on, the more "chancey" your prediction will be. As it turns out there's a quick way to get a measure of how valid your correlation factor is under different data conditions. (As a general rule, if you don't have much data - unless your correlation factor is quite close to plus or minus one - you can't be too sure of it).

One procedure for a quick check on the validity of your correlation coefficient is as follows :

a) Decide how sure or valid you'd like (or need) the correlation coefficient to be - say 95 %

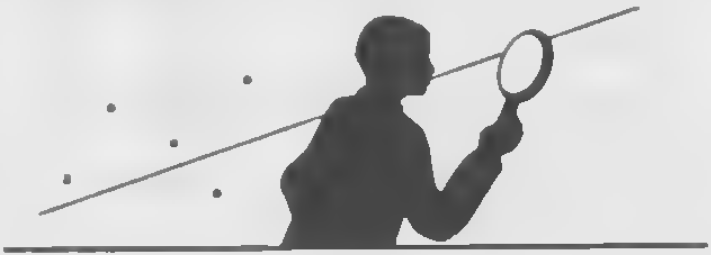
b) Locate the r_{test} (test correlation coefficient) value from the table at the end of this chapter - for the degree of certainty you've selected, and the number of samples you have to work with - (don't worry about the "degrees of freedom" column in the table for now)

c) If your calculated correlation coefficient is *greater than* r_{test}, you can be certain (to the degree selected) that your straight line approximation is valid

In our case, the calculated correlation coefficient is 0.99. We compare this to the r_{test} value, at 95 % certainty for 5 samples (find this value in the tables)

$$r_{test} = .878$$

Since our correlation coefficient is greater than r_{test}, we can assume that our correlation coefficient is valid, to a 95 % degree of certainty. (Being 95 % certain of a conclusion means that 95 times out of 100 you will be correct).



STOCK DIVIDEND PROJECTIONS (Trend Line Analysis)

II·2

You'll find many instances when your data is collected in the form of a series of yearly figures - and your job is to predict what will happen in years to come. This type of prediction involves what statisticians call "trend line analysis" - which is really just a special type of linear regression. Your calculator has features that make trend line analysis easy.

Example : A stock that you've been keeping your eye on has reported the following earnings per share during the past few years :

£ 1.52 in 1972
1.36 in 1973
1.53 in 1974
2.17 in 1975
3.60 in 1976

You'd like to predict the earnings per share for the next three years. You'd also like to know in what year you could expect the earnings per share to reach £6.50.

Target : You wish to enter the data you *have* into your calculator, and then use trend line analysis to make predictions. You'd also like some feeling as to how well the two sets of data are correlated.

Tools : First, you'll enter your data, using the [x,y] and [Σ+] keys. In this case the "x" values are a series of years *in sequence*, and the "y" values are the stock dividends recorded for each year. (Data for a series of successive years is common for trend line analysis situations.)

Now - here's an important feature - for Trend Line Analysis your calculator will *automatically* add 1 to the x variable for you. This means that:

You can enter the first x value (say the first year, 1972) and press [x,y], then enter a y value (say £1.52 earnings per share) and press [Σ+]. The first data point is entered.

Then :

You can enter the second data point by just entering the y value (in our case £1.36) and pushing [Σ+]. The calculator will automatically handle the x variable for you - incrementing it by 1.

This will come in handy whenever you're analysing data from successive years - or whenever your x variable is going up in increments of 1.

After your data is entered :

To make predictions on earnings for future years : just enter the year and press 2nd y'

To predict in what year a certain level of earnings per share will be reached
Enter the earnings and press 2nd x'

To see how well the two sets of data correlate Press 2nd r

To check on the validity of the correlation - compare your correlation factor to the "r_{crit}" value in the table at the end of this chapter.

Keying It In :

Press		Display/Comments	
2nd CA		0.	This clears the display and memories
2nd FIX	2	0.00	This sets the display to read out only two decimal places
Now enter your data			
1972 x⇄y	1 52 Σ+	1.00	Note : the calculator will
	1 35 Σ+	2.00	increase the value by 1
	1 53 Σ+	3.00	automatically unless
	2 17 Σ+	4.00	another value is entered
	3 60 Σ+	5.00	The years are automatically "passing by" for you !

Decision Time : Now, to predict the earnings for future years (future y values) just key in the year, and press 2nd y'

1977 2nd y'		3.53	Dividends of £3.53 per share are projected for 1977
1978 2nd y'		4.03	for 1978
1979 2nd y'		4.52	for 1979

You can now make decisions based on the pattern of growth you're watching or go on to predict when the earnings per share will reach a specified value For example, to calculate when the earnings will reach £8.50 (if the earning trend continues) - you just enter the 6 50 and press 2nd x'

6 50 2nd x'		1952.97	or about 1983.

Going Further : If you'd like to see how well the two sets of data are correlated just press ·

2nd CORR 0.85

You can get an idea as to how "valid" the correlation coefficient is by checking in the table at the end of the chapter. First, find the line with the same number of samples you have here (5) Now - scan across to the right at the "$r_{.05}$" values and find the *first one that's larger* than your r value (85). (You should find the value .878) You can now glance up to the certainty values at the top of the table to draw a conclusion you can be about 90-95 % sure that this correlation coefficient is "valid"

In this example we'll use the linear regression feature of your calculator, in particular the *correlation* feature (2nd CORR) to help make a decision on *whether or not two variables are related.* It may often appear that one factor in your business life is related to another - but just how closely they really "track" is often unclear With your calculator you can get a more accurate picture of just how much relation there is between two variables.

Example : Test Scores vs Performance

Let's say your sales manager is spending a considerable sum on a test for prospective sales employees. You'd like to see if this test is actually telling you anything about how well the employee will function in the field. Does a higher test score mean superior sales performance ? How strong a *correlation* is there between these two factors in your business ?

Let's say you have samples of the test scores for 10 employees, along with records on sales performance expressed as the percentage of the time that each employee exceeded his or her weekly sales goals last year. The data is tabulated below

Employee	Employee Test Score (x)	Employee Sales Performance (y)
Jarry	5	10
Ross	13	30
Joe	6	30
Ralph	10	40
Mary	15	60
Gary	20	50
Dean	4	20
Carole	16	60
Ted	18	50
Alice	6	20

RELATING JOB PERFORMANCE TO TEST SCORE (Establishing correlation)

II-2

Target : Determine if there is a genuine relationship between test scores and sales performance. If so, what is the relationship, and can you get a feel for how reliable it is ?

Tools . Your calculator's linear regression feature can easily apply some high powered statistical mathematics to this problem for you

First . Enter your data with the [x,y] and [Σ+] keys.

Then Study the correlation coefficient (r) by pressing [2nd] [CORR], and consulting the "r_{test}" table at the end of this chapter.

Keying It In . Enter the data and determine the correlation coefficient

Press	Display/Comments	
[2nd] [CA]	0	Clear entire machine including memories
[2nd] [FIX] 2	0.00	Set display to read-out to 2 decimal places
5 [x,y] 10 [Σ+]	1.00	
13 [x,y] 30 [Σ+]	2.00	
8 [x,y] 30 [Σ+]	3.00	
10 [x,y] 40 [Σ+]	4.00	
15 [x,y] 60 [Σ+]	5.00	
20 [x,y] 50 [Σ+]	6.00	
4 [x,y] 20 [Σ+]	7.00	
16 [x,y] 60 [Σ+]	8.00	
18 [x,y] 50 [Σ+]	9.00	
6 [x,y] 20 [Σ+]	10.00	
		To find the correlation factor
[2nd] [CORR]	0.87	= r

Decision Time :

The correlation factor of 0.87 tells you that there is a pretty good relationship between the test scores and the indicator for employee performance that you're using.

To get a general feel for how valid this correlation factor is - glance at the table at the end of this chapter

RELATING JOB PERFORMANCE TO TEST SCORE

II-2

Find the line for the number of samples you've got (in this case 10) and examine the "r_{test}" values listed to the right. Your value for r (the correlation coefficient - 0.87) falls between .765 and .872 listed on the table - so you can be between 99 % and 99.9 % sure its a "valid" correlation coefficient - there is a definite relationship between these variables

Going Further : Future Predictions

Using the data you've got in your calculator, you can now go on and predict employee performance for any given test score. Just key in the score (x) value and press 2nd $\hat{y}$. Some examples

Press	Display/Comments
7 2nd $\hat{y}$	24.92
25 2nd $\hat{y}$	73.23
30 2nd $\hat{y}$	86.65

If you wish to make *future predictions* again at some later data, you can easily write down the *equation of the line* your calculator has "drawn" through your data using the 2nd LRR and 2nd INTC key sequences.

2nd LRR	2.68	slope value (m)
2nd INTC	6.14	intercept value (b)

The equation of any straight line can be expressed as : y = mx + b.
y = (Slope) x (x) + (Intcp) so in this case the line is given by y = 2.68x + 6.14

So, if at some future data you wish to make a prediction - you only need note the slope and intercept values. If an employee then scores a 24.2 on his test, you can substitute that result for x in the equation for the line to predict his or her performance.

2nd CA	0	Clear machine
2.68 x 24.2		
+ 6.14 =	70.996	-a good prospect for field sales !

How to Use "r_{test}" Table for Correlation Coefficients

Find the number of samples you have in the left hand column, and scan across to the right - comparing the values of r_{test} listed in the table to your calculated correlation coefficient. Find the values of r_{test} that your correlation coefficient fits "in between" and scan upward to read the " degree of certainty" limits for your coefficient. If your correlation coefficient is too small for you to find in the table, then you're less than 80 % sure of its validity.

The values in this table are from the formula :

$$r_{test} = \left(\frac{t^2}{t^2 + df}\right)^{1/2}$$

where df = the degrees of freedom, and t is the t value for df from table C in the *Appendix*. (Chapter II 5)

Example : For 15 samples, a correlation coefficient of .525 can be considered between 95 % and 99 % "valid".

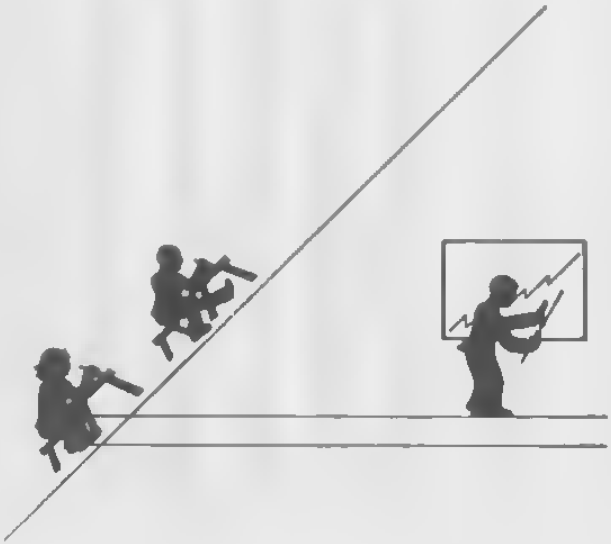
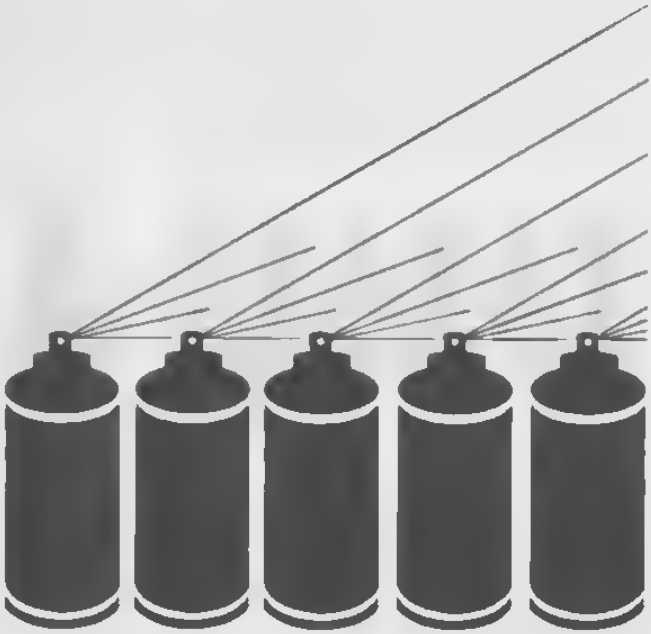


Table of "r_{test}" Values - Test Values for Correlation Coefficient

# of Samples	(df) degrees of Freedom	80 %	90 %	95 %	99 %	99.9 %
3	1	0.951	.988	.997	1.000	1 000
4	2	0.800	.900	.950	.990	999
5	3	0 687	805	878	.959	991
5	4	0 608	729	.811	.917	974
7	5	0.551	.669	768	.875	951
8	6	0.507	621	.707	.834	925
9	7	0.472	582	666	798	898
10	5	0.443	.549	632	765	872
11	9	0 419	.521	.602	.735	847
12	10	0 398	.479	.576	.708	823
13	11	0 380	475	.553	.684	801
14	12	0 365	457	.532	661	.780
15	13	0.351	441	.514	.641	.760
16	14	0.338	.426	497	.623	.742
17	15	0.327	.412	.482	.606	.725
18	16	0.317	.400	.468	.590	.708
19	17	0.308	389	.455	.575	693
20	15	0 299	378	444	.561	579
21	19	0 291	.369	433	.549	665
22	20	0 284	360	423	.537	652
23	21	0.277	352	413	.526	640
24	22	0.271	344	404	.515	.529
25	23	0.265	.337	.396	.505	618
26	24	0 260	330	388	.496	607
27	25	0.255	323	381	487	597
28	26	0.250	.317	.374	.479	588
29	27	0.245	.311	.367	.471	579
30	28	0.241	306	361	463	570
31	29	0.237	301	.355	.456	.562
32	30	0.233	296	.349	449	554
42	40	0 202	.257	304	393	490
62	60	0.165	.211	.250	.325	405
122	120	0.117	.150	.178	.232	.294

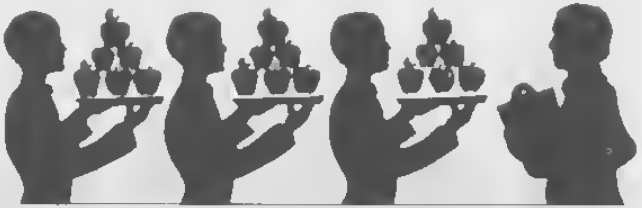
Testing Claims



Many times in your business (or everyday life) you're forced into making decisions (buy/not buy - accept/not accept) about a *large lot* or *quantity* of items. Time and expense usually allow you only to examine and test a few *samples* of the large population you have to decide on. (This is often the case in an "incoming quality control" operation, for example).

Whenever you're in this situation - deciding about a large *population* based on a smaller *sample* - a certain amount of *uncertainty* is always present. The sample is giving you some information to be sure - the key is using your sample data wisely. When a manufacturer *claims* that a lot of goods meets a certain specification - data from your sample can be used to test that claim to a *specified degree of certainty*. The examples in this chapter are designed to show you how - and how your calculator can help.

In this chapter we'll get into the examination and analysis of data from samples, and the how and whys of relating that data to larger populations. We'll proceed step-by-step through the analysis of several typical case situations with some sample data - so that you will hopefully be able to apply the tools illustrated to your own analysis situations. Some fairly "high powered" statistical methods are involved here, but with your calculator keeping tabs on the mathematics, you'll be surprised at how the implementations of these techniques get simplified.



First Things First

In most of the examples we'll be considering in this chapter the following situation is addressed:

A manufacturer (grower/supplier, etc.) makes a claim about a particular specification for a shipment of goods he's just delivered. This *claim* usually is expressed as a *mean* value for the population

"The mean weight of product in these containers is 510 grams."
"The mean lifetime of these batteries under standard load conditions is 180 hours . . ".

You usually get a chance to test a sample of these parts to see if they are O.K.

The first thing to do is to take as large a sample as possible and examine the mean value of the specification for the sample, as well as its standard deviation. Your Advanced Professional Calculator has keys that make this quite easy. Just take your measured sample data and enter it with the [Σ+] key. The [2nd] [MEAN] and [2nd] [Sdev] key sequences will give you the mean and standard deviation of your sample data and a "first step" in your decision. Is the mean close to the claimed value ? Is the standard deviation large or small ?

A large standard deviation indicates a *highly varying* value for the parameter you're examining - and may be enough reason for you to reject the shipment immediately ! The rest of this chapter tells you "how to use statistical inferences" in calculator decision making based on your sample results. Focus on these important concepts . the *population* refers to the entire set of items being tested, the *sample* is a part of the population that's been "picked out" for test. You'll be making decisions about the *population* based on *sample* data, and the *level of certainty* you decide.

MEAN WEIGHT OF AEROSOL DISPENSERS

II-3

Testing the manufacturer's claim with concern about *both* upper and lower limits.

Here's an example situation that calls for a decision about a population based on a sample. Let's say a large shipment (population) of aerosol cans of insecticide has just arrived at your receiving dock. The manufacturer claims that the cans contain, on the average, 510 grams of insecticide each. Maybe you usually just take this fact at face value - but this time you'd like to be sure that he's meeting this claim.

You're concerned about this problem for two reasons - these particular cans don't work properly if they're overfull, and you're getting gypped if they're less than full. The ideal case is when each can contains exactly 510 grams - and you're concerned about the manufacturer meeting this "spec" - *both on the high and low end* (This is what's called a "two sided" or "two-tailed" decision making process).

You have a technician measure the weight of 40 cans (the sample) and tabulate the data for you. With a quick calculation on your calculator you found:

The mean sample weight is 508.75 g (usually labeled $\bar{x}$)

The sample standard deviation (labeled s_x) is 19.97 g

The decision - is the manufacturer meeting his claim? Should you accept the shipment or reject it? Can the sample data give you a little more to go on? It can - read on!

Target: Let's say you want to be 95 % sure that the manufacturer has not met his claim before you reject the shipment. Your target here is to get as much information as you can about the population, based on the data you have from the sample.

Tools: Here your *sample size* is over 30 items - which statisticians generally agree to as an informal boundary between "large" and "small" samples. For your "large" sample of 40 items you may assume that the *sample standard deviation* (s_x) is a pretty good estimate of *the population standard deviation* (usually labeled with the lower case Greek letter sigma, σ).

This fact often allows you to immediately reach some important conclusions. Most manufacturing processes deviate from the specified or target value in a "normal" way. This means that the population values can often be considered to follow the normal curve. If this is the case, then about 95 % of the cans will be within ± 2 standard deviations of the mean.

The sample standard deviation of 19.97 implies a range of ± 2 (19.97) (± about 40 grams) for about 95 % of the cans. If, in your case, a ± 40 grams variation in the weight of the cans is by itself unacceptable, you may need to reject the cans based on this standard deviation value alone.

If the standard deviation value *is* acceptable to you - you now need to proceed to a little more complete analysis. There's a tool from statistics that lets you :

a) Select a degree of certainty for your decision to accept or reject - say 95 %

b) With a straightforward calculation you can now establish a *range* within which the population mean (labeled μ) lies, to the degree of certainty you selected. The formula for this range is .

$$\text{Range for } \mu \text{ at degree of certainty you select} = \bar{x} \pm \frac{\sigma}{\sqrt{n}} Z$$

In this formula $\bar{x}$ is your sample mean, n is the number of samples and z is the "z score" for the degree of certainty you select. This "z score" is found in Table A in the *Appendix* (Chapter II-6) - from column II where z values for checking *both* upper and lower levels are tabulated. If you check in that table - column II reads a z value of 1.96 at 95 % degree of certainty.

So summarizing .

From Table A $z = 1.96$

$\sigma = s_x = 19.97$ (for large samples only, $n > 30$)

$n = 40$

$\bar{x} = 508.75$

and you need to

Evaluate $\bar{x} \pm \frac{\sigma}{\sqrt{n}} z$

Keying It In : A good way to begin this calculation is to evaluate the last term $\frac{\sigma}{\sqrt{n}}z$, and store it in memory 1

Press	Display/Comments
2nd CA	0. Clear all memories and registers
2nd FIX 2	0.00 Set calculator to display only 2 decimal places
19.97 ÷ 40 √x ×	This evaluates $\frac{\sigma}{\sqrt{n}}z$
1.96 = STO 1	6.19 and stores it
	Next evaluate $\bar{x}+\frac{\sigma}{\sqrt{n}}z$
+ 508.75 =	514.94
	Evaluate $\bar{x}-\frac{\sigma}{\sqrt{n}}z$
508.75 − RCL 1 =	502.56

Manufacturer's claimed value of 510 g falls inside these limits - accept !

Your *sample* is telling you that the *population* mean is somewhere between these two numbers, with 95 % certainty.

Decision Time : You now have a better picture of what your sample is telling you about the shipment. You've got two values, 502.56 grams and 514.94 grams, and now you can say with 95 % certainty that the mean weight value for the shipment (the whole population) lies in between these two values. Since the manufacturer's claimed weight value of 510 grams falls within these limits, as far as you can tell from your sample, he's met his claim. Based on this analysis you'd *accept* the shipment of aerosol cans.

The analysis you've just done is summarized for you here :

e) First, get as large a sample as possible and measure it - calculate the sample mean ($\bar{x}$) and standard deviation (s_x).

b) Decide on the degree of certainty you need and calculate the *predicted range for the population mean* with the formula below :

$$(\text{Range for } \mu) = \bar{x} \pm \frac{\sigma}{\sqrt{n}} z$$

(Remember you find z from column II in Table A for the degree of certainty you select). For samples with over 30 items, you can approximate σ with s_x.

c) If the manufacturer's claim value falls *inside* the range, accept and vice versa.

Further Notes : When selecting the degree of certainty for a problem, it is important to realise how the statistical process works. The amount of information you have in your sample does not change. If you select a very high degree of certainty, then what you are certain about is less definite. (Got that ?).

Here's an example : A mechanic looks at your car and tells you that he is pretty sure that it will cost about £90 to £100 to fix it. If you tell him that he has to be 99.9 % sure of his estimate, he will probably estimate a wider range, say £50 to £200. If the situation you are investigating demands more certainty about a smaller range, then you may need to take a larger sample.

Testing a manufacturer's claim - with concern about meeting minimum specifications only.

In this example let's say you're manufacturing an electronic product into which you put a battery. A manufacturer has just shipped you 5000 of them, and he claims the mean lifetime for this shipment, (population) is 180 hours. In this case, you want to check on the manufacturer's claim, but *what's critical to your decision* to accept the shipment is that the mean lifetime *of* the shipment of batteries is (as near as you can tell) no *less* than 180 hours. You don't really care if the batteries have longer than 180 hour life. (In fact, this would make you very happy). You're really just concerned about checking the "low side" of their performance. (This is what's called a "one-sided" or "one-tailed" decision process).

To test the population of 5000 (N), you have a technician select a sample (n) of 100 batteries and measure their average lifetime under standard load conditions. (Since this test ruins the batteries - you decide you can't afford a much larger sample than 100 items.) Your technician finds out that the sample mean lifetime ($\bar{x}$) is 175 hours, with a sample standard deviation (s_x) of 18 hours. Your decision ... accept or reject the shipment?

Actually - you already have quite a bit of information to go on. First of all, since your sample of 100 batteries qualifies as a "large" one ($n > 30$), the *sample* standard deviation (s_x) is considered to be equal to the *population* standard deviation (σ). So you really have an immediate decision to make ... is the standard deviation of the shipment acceptable to you? In this case, σ = 18 hours. Let's say that you *can* accept this variability in the shipment. Now you need to make a judgment about the *population* mean (μ). Your sample mean ($\bar{x}$) is 175 hours. How can you use this information to draw a conclusion about the population mean lifetime?

Target: You need to make a decision about whether or not to accept the shipment, based on the sample data, and let's say you want to be *95 % certain* that you don't reject good batteries. Your primary concern is that the battery life be *not much less than* 180 hours - if the average life is longer than this, fine!

Tools: There's a formula from statistics that allows you to calculate, from your sample data a *range* in which the population mean will lie. With this *range* you know, based on your sample data and degree of certainty you select, an upper and a lower limit for the *actual* population mean. The formula is

$$\text{Range for population mean} = \bar{x} \pm \left[\frac{(N-n)}{(N-1)}\right]^{1/2} \frac{\sigma}{\sqrt{n}} z$$

(This formula may look complex - but it's easy to evaluate on your calculator).

In this case x is the sample mean lifetime = 175 hours
N is the population size = 5000
n is the sample size = 100
σ is the standard deviation of the population, which in this case can be approximated by s_x (= 18 hours)
and
z is the z value found from Appendix Table A (Chapter II-6), for the degree of certainty you select (here 95 %), taken from column I - since you will reject based on only one boundary in this case (This is called a "one-sided" or "one-tailed" test).
In our case z = 1.65

A note here : In this formula the expression $\left[\frac{(N-n)}{(N-1)}\right]^{\frac{1}{2}}$

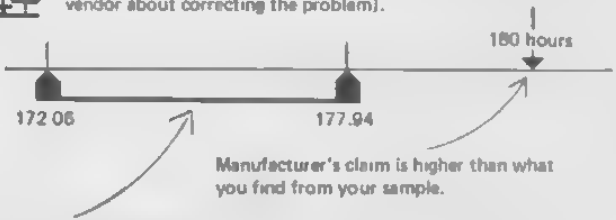
is a factor which allows for the fact that when you test the batteries in the sample, you *remove* them from the population and can't return them after the test. This removal of sample items strictly speaking affects the "randomness" of your selection - and this "factor" corrects for this fact.

Keying it in . In doing this calculation, first evaluate the quantity.

$\left[\frac{(N-n)}{(N-1)}\right]^{\frac{1}{2}} \frac{\sigma}{\sqrt{n}}$ z and store it. Then go on to complete the calculation.

Press	Display/Comments	
2nd CA	0	Clear entire calculator
2nd FIX 2	0.00	Set calculator to display 2 decimal places
((5000 − 100)	4900.00	
÷ (5000 − 1)	4999.00	
(√x (X 18 ÷ 100		
√x X 1.65 = STO 1	2.94	Now add $\bar{x}$
+ 175 =	177.94	Upper limit
175 − RCL 1 =	172.06	Lower limit

Decision Time. Here you are predicting that the population mean actually has a value somewhere between 172.06 and 177.94 - and what you really want to focus your attention on here is that you now know from your sample (with 95 % certainty) that the population mean is *not greater than 177.94*. So, based on your sample data, the battery mean lifetime *is* less than 180 hours, and based on this analysis you'd reject the shipment (or talk with your vendor about correcting the problem).



CHECKING ON TINT IN PAINT MIX

11·3

Testing a claim using data from a *small sample* ; with concern about *both* upper and lower limits.

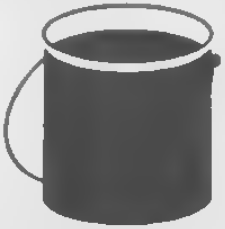
In this example you're doing your own check on a formulating process in a paint manufacturing operation. Specifically - you're checking on the amount of red dye being mixed into 5 gallon containers of "rose" coloured paint. The process specification calls for 15.5 ounces of red tint in each can. You select a random sample of 8 cans, and through analysis, find the tint content to be :

15.2 oz	15.8 oz
15.0 oz	16.1 oz
15.7 oz	15.6 oz
15.9 oz	15.9 oz

(Let's say the analysis is expensive - so you're limited to this small sample quantity). Your decision in this case - should you stop the manufacturing and adjust the process, or are things O.K ?

Target - You want to get as much information as you can about the population mean for the amount of red tint, based on data from the small sample you have to work with. To do this you can use a statistical technique especially designed to handle the "small sample" situation. This technique allows you to calculate a *predicted range* of values that the population mean (μ) will fall into, with a degree of certainty you select.

This predicted range of values can form the basis for your decision. If your calculated range of values *includes* the specification value of 15.5 oz you don't have enough indication of trouble to "stop the line". If the range of values you calculate from your sample data does *not* include your specification value, however, you can be sure (to the degree of certainty you select) that you've got a problem and an adjustment should be made. Also, note in this case that you're concerned about both "limits" on the amount of tint - too much will give you a colour that's too red, while too little tint will provide too weak a colour.



Tools : Since your sample size in this case is *less than 30,* it falls into the "small" sample category and some statistical methods especially suited to this situation should be used. To use these tools :

First, decide on a degree of certainty you need for the decision– let's say 90 % in this case. Then, calculate the predicted range for the mean tint (population value) using the formula below .

$$\text{Predicted Range for Population Mean} = \bar{x} \pm \frac{s_x}{\sqrt{n}} t$$

where $\bar{x}$ is the mean value for your sample

s_x is the sample standard deviation

n is the size of the sample

and

t is a value found from Table C in the Appendix (Chapter II-6)

- for the degree of certainty you select (90 %)
- and the number of *degrees of freedom* for the problem, (df).

  In this case :

  df = n – 1 = 7.
- checking in Table C - you'll find a t value of 1.895

To find the sample mean ($\bar{x}$) and sample standard deviation (s_x), you can use special keys on your calculator

Keying It In : First, clear your machine and enter the sample data with the [Σ+] key :

Press		Display/Comments	
[2nd] [CA]		0	Clear entire machine
[2nd] [FIX]	2	0.00	Set display to read out 2 decimal places
15.2 [Σ+]		1.00	Display keeps track of
15.0 [Σ+]		2.00	the number of data
15.7 [Σ+]		3.00	entries.
15.9 [Σ+]		4 00	
15 8 [Σ+]		5.00	
16.1 [Σ+]		6.00	
15 6 [Σ+]		7 00	
15.9 [Σ+]		8.00	

Now you can, with only a couple of keystrokes, calculate the sample mean and standard deviation :

2nd MEAN	15.65 The sample mean, $\bar{x}$
2nd S.DEV	0.37 The sample standard deviation, s_x

At this point you already have quite a bit of information. The sample mean looks "pretty close" to 15.5, and the standard deviation is low - indicating that there's a relatively low "spread" to your measured sample red tint values. But remember - your sample is a small one - and you need to make an important decision about a much larger population based on it. This is where the statistical method can be helpful. Now go on to calculate the predicted range of the population mean (μ).

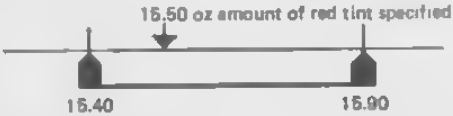
Predicted Range for $\mu = \bar{x} \pm \frac{s_x}{\sqrt{n}} t$

Now you know that $\bar{x} = 15.65$ $n = 8$
$s_x = 0.37$ $t = 1.895$

Begin by calculating $\frac{s_x}{\sqrt{n}} t$

Press	Display/Comments
2nd CA	0 Clear entire machine
2nd FIX 2	0.00 Set display to 2 decimal places
0.37 ÷ 8 √x × 1.895 = STO 1	0.25 Now add this to x to find upper range limit
+ 15.65 =	15.90 $= \bar{x} + \frac{s_x}{\sqrt{n}} t$
15.65 − RCL 1 =	15.40 $= \bar{x} - \frac{s_x}{\sqrt{n}} t$

Decision Time :



Your sample states that with 90 % certainty the population mean lies between these two values

From your small sample of 8 cans you can state that with 90 % certainty the *population mean* value for the red tint is between 15 4 and 15.9 oz. Since your specified value of 15 5 lies in between these limits, the process appears to be O.K. ! (You don't have enough data to call a shut-down as yet !).

Testing a claim using data from a *small sample*, with concern about maximum specification only.

In this example let's say you're called in to help out the buyer for a large chain of drugstores. A large shipment of *Dr. Sam's Cough Medicine and Elixir of Life* has just arrived. The manufacturer claims that the preparations contains 8 % alcohol. The buyer needs to be certain that the population's mean alcohol content is no greater than 8 %. He can only get data on a small sample : 5 bottles were selected at random and analysed. The bottles showed 7.85 %, 8.33 %, 7.97 %, 8.31 % and 7.76 % alcohol upon test. Should you advise the buyer to reject the shipment ? He tells you he'd like to be 95 % sure of his decision.

Target : In this case you need to find out all you can about the population mean (μ) from the small sample. Your primary concern is that the mean alcohol content of the shipment is not over 8 % before accepting it.

Tools : In this case you're dealing with a small sample ($n < 30$), so you should use the statistical analysis method suitable for small sample analysis as outlined below.

First, using your calculator, enter the sample data with the [Σ+] key, and calculate the sample mean ($\bar{x}$) and sample standard deviation (s_x) with the [2nd] [MEAN] and [2nd] [SDEV] key sequences. Next, using the formula below, calculate the predicted range for the population mean.

Predicted range for the population mean $= \bar{x} \pm \frac{s_x}{\sqrt{n}} t$

In this formula

x is the sample mean,

s_x the sample standard deviation

n the number of items in the sample (5)

t the "t" value you find in the Appendix (Chapter II-6)

In this case the t value is found in *Table B*, because you are concerned with only one limit. In this table, locate the t value for the degree of certainty you require (here 95 %) and the number of degrees of freedom, (df) equal to $n - 1 = 4$. (You should find at value of 2,132).

If you can be 95 % sure that the alcoholic content of the "Elixir" is greater than 8 %, you plan to reject the shipment.

CHECKING PHARMACEUTICAL SPECIFICATIONS

Keying It In : First, enter your data using the [Σ+] key, and calculate the sample mean and standard deviation values

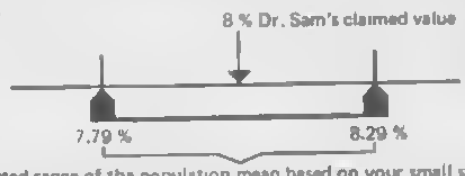
Press	Display/Comments
[2nd] [CA]	0 Clear the entire machine
[2nd] [FIX] 2	0.00 Set display to 2 decimal places
7.85 [Σ+]	1.00 Enter your data : the
8.33 [Σ+]	2.00 calculator keeps count
7.97 [Σ+]	3.00 of the number of entered
8.31 [Σ+]	4.00 data points
7.76 [Σ+]	5.00
[2nd] [MEAN]	8.04 the sample mean ($\bar{x}$)
[2nd] [S.DEV]	0.25 the sample standard deviation (s_x)

Now clear the calculator and calculate the predicted range of the population mean

First calculate $\frac{s_x}{\sqrt{n}} t$ and store it, then calculate $\bar{x} \pm \frac{s_x}{\sqrt{n}} t$.

[2nd] [CA]	0 Clears everything
[2nd] [FIX] 2	0.00 Set decimal to 2
0.26 [÷] 5 [√x] [×] 2.132	
[=] [STO] 1	0.25 Now add $\bar{x}$ to calculate $\bar{x} + \frac{s_x}{\sqrt{n}} t$
[+] 8.04 [=]	8.29 upper limit Now calculate $\bar{x} - \frac{s_x}{\sqrt{n}} t$
8.04 [−] [RCL] 1 [=]	7.79 lower limit

Decision Time :



Predicted range of the population mean based on your small sample.

Based on this analysis you'd accept the shipment. As far as you can tell, based on the small number of samples you've tested, the actual amount of alcohol may be as low as 7.79 %. Since Dr. Sam's claimed value is 8 % - you have no argument with the shipment. In this case the entire predicted range of the population mean would have to be greater than 8 % before you'd reject the shipment with 95 % certainty.

CHECKING CLAIMS ON DEFECTIVE PARTS

Checking on a proportion of defective parts - with concern about maximum percentage defective only.

In this "case history" you are called in to aid the manufacturer of *Elflasho* flashlights. He's just received his first shipment of flashlight bulbs from a new manufacturer - and wants to be particularly sure he's got a good shipment before accepting it. Testing the parts is quite simple in this case - they either light or they don't - so a sizeable sample can be easily tested. The new bulb manufacturer, *Brite Spot Systems, Ltd.* insists that the shipment (population) will contain no more than 12 % defective bulbs.

The *Elflasho* line foreman has 250 of the bulbs tested, and of these, 43 fail (17.2%). He asks your advice - should he accept or reject the shipment based on this data. He'd like to be 90 % sure the lot has more than 12 % defective bulbs, before he rejects the shipment and looks for a new vendor.

Target : In this case you're dealing with a claim about a *proportion*, and so you should use a statistical technique especially suited to handling the problem.

First, you use the formula below to calculate the *predicted range of the population mean*, as in previous examples. In this case, however, instead of the population mean being a numerical value (such as weight, or % volume) it's the *proportion of defective parts in the population.*

The formula for the range in this case is :

$$\text{Predicted Range of the Population Mean Proportion} = \bar{P} \pm \left(\frac{\bar{P}(1 - \bar{P})}{n} \right)^{1/2} z.$$

where - $\bar{P}$ is the proportion of defective parts found in the sample

(In this case $\frac{43}{250}$ or 0.172)

n is the sample size (250)

and

z is the z value found from *Table A* in the *Appendix* (Chapter II-6).

You're concerned with one limit here (you should reject if the shipment is over 12 % defective, and accept otherwise). Since you wish to be 90 % sure of the reject decision, the z value from *Table A* is found from column 1 to be 1.28.

Once you've calculated a range for the population mean, you'll compare it to the manufacturer's claim - and make your decision.

Keying It In : You already know that the proportion of defective parts in the sample is 17.2 %, or 0.172. Now evaluate the expressions

$$\bar{P} + \left(\frac{\bar{P}(1-\bar{P})}{n}\right)^{1/2} z \text{ and } \bar{P} - \left(\frac{\bar{P}(1-\bar{P})}{n}\right)^{1/2} z$$

to calculate the predicted range of the mean.

Begin by evaluating $\left(\frac{\bar{P}(1-\bar{P})}{n}\right)^{1/2} z$, and storing it.

By looking ahead, you can see this problem needs to be solved twice. Storing the first key sequence in program memory allows you to change variables, such as z (degree of certainty), and repeat problem solution with just a few keystrokes.

First, key the problem into program memory assuming P is stored in memory 0, n in memory 1 and z in memory 2.

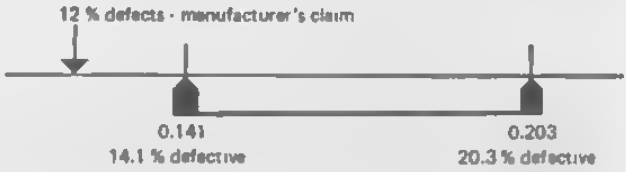
Press	Display/Comments	
2nd CA	0	Clears entire machine. Use 2nd RST when clear all is not desirable
2nd LRN	00 00	Calculator in learn mode
RCL 0 X 1 − RCL 0	09 00	First R/S stops
+ RCL 1 = √x X	15 00	program to display
RCL 2 = STO 3	20 00	upper limit and second
+ RCL 0 = 2nd R/S	25 00	R/S displays lower limit
RCL 0 − RCL 3 = 2nd R/S	0	Since the last R/S is at location 31, the calculator automatically leaves the learn mode.

Now enter the known values into data memories and run the program.

Press	Display/Comments	
172 STO 0	0.172	Store P in memory 0
250 STO 1	250.	Store N in memory 1
1.28 STO 2	1.28	store z in memory 2
2nd R/S 2nd FIX 3	1.280	Reset program to step 00 and display to 3 places.
2nd R/S	0.203	Upper limit
2nd R/S	0.141	Lower limit

NOTE . Do not clear or turn the calculator off.

Decision Time ·



Predicted range for the mean percentage of defective parts in the population.

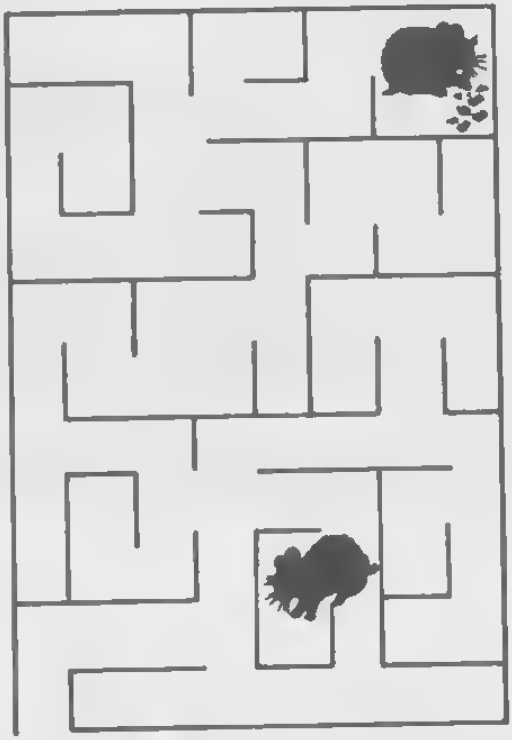
In this case your sample is telling you that the lowest expected percentage of defective parts is 14.1 %. You're 90 % sure that the manufacturer is *not* living up to his claim and *Elflasho*'s needs based on this analysis, you advise their foreman to reject the shipment.

Going Further : As it turns out, the foreman at *Elflasho* is not immediately ready to ship back the bulbs. (It seems the president of *Brite Spot Systems* is also the son-in-law of Mr. Elflasho). He needs to be *very* sure. You can re-check the decision at a higher degree of certainty quite easily. Let's say you both agree that if he's 95 % sure the shipment is bad - it will go back and hang the consequences. First locate the z score in Table A for a 95 % degree of certainty. Store the new value for z in memory 2 and rerun the program.

Press	Display/Comments	
1.65 STO 2	**1.650**	Store new z in memory 2
2nd RST	**1.650**	Reset program to step 00.
2nd R/S	**0.211**	Upper limit
2nd R/S	**0.133**	Lower limit

Answer at 95 % certainty - you'd *still* reject the shipment !

Testing for Change



There are a variety of situations we're all in from time to time, where *decisions concerning change* are involved. In these situations you need to decide whether or not some new process, method, policy, etc. has created some genuine change over an old one. Situations such as this may arise when trying new educational techniques production methods, engineering systems etc.

In some situations a change may appear overwhelming, and it's an "open and shut" case that something is clearly different. In some other cases, however, it may appear that some improvement has been made - but it's not an overwhelming change. Here's where decision-making becomes more difficult A decision to endorse or institute a new procedure or process in such a situation, based on data from small samples, can be a pretty tricky business.

Several methods from statistics are available to aid in the study of change. These methods involve some fairly sophisticated techniques - but as before we'll try to boil the *use* of them down to a series of step-by-step procedures. Your advanced professional calculator - with AOS and parentheses helping out - will be a powerful ally here. (In fact, *without* your calculator handling the arithmetic involved, using these techniques would be quite a bother).

The specific statistical method we'll be focusing on in this chapter may be called a confidence interval procedure. This procedure often works together with another one called the "F-test". These two procedures enable you to decide - with a degree of certainty that *you* select - whether or not a genuine difference exists between one set of data and another (assuming that the populations are, at least, approximately normal). In cases where your decision involves a considerable sum of money - these tests can be an important part of your decision-making process. In cases where lives may be involved - such as in pharmaceutical research or immunology - these procedures can be crucial !

This chapter uses "confidence interval" and "F-test" methods in analysing two "case histories" where a decision must be made based on whether or not a change or difference exists between the results of two processes.

Uncorrected "confidence interval" method for analysing change.

Let's consider a case where you're "called in" to help out with a decision on an oil pipeline. A new pipe supplier on the scene (Apex) claims that a new "neverust" coating process on his company's product will provide "up to three times longer life" over standard, noncoated pipe. The decision to change to the new pipe will involve a significant unit cost increase, and your pipeline requires several hundred miles of pipe - so you need to be pretty sure (let's say 95 %) about any decision made.

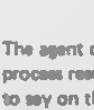
You ask to see the *data* which supports Apex's claim of up to three times longer life. As it turns out, all of the data that Apex has are the results of six experiments. In each experiment a length of standard pipe and a length of coated pipe were buried side by side (in six different locations), and the weight loss due to corrosion was measured (in ounces per foot per year). The results of their tests are tabulated below :

APEX NEVERUST PIPE CO.
TEST DATA
(yearly weight loss in ounces/foot/year)

Uncoated Steel Pipe	Apex Neverust Coated Pipe
3.66	2.66
1.28	0.45
1.84	0.92
3.66	1.69
1.83	0.06
6.00	0.16

The agent claims that from this data you can "clearly see" that Apex's new coating process results in pipe that lasts "up to three times longer". He has nothing more to say on the matter - so you tell him you'd like to think about it.

Target : Your goal in this case is to determine just how much you know about the pipe's performance, based on only the sample. Since the sample (6 coated & 6 uncoated pipes) is small, methods of "statistical inference"will be important here.



What you really need to do is to predict what the mean difference in yearly weight loss would be between a coated and an uncoated pipeline, based on the experimental data from Apex (the sample) at a 95 % degree of certainty.

Tools : First, you can take a "statistical look" at the sample data by examining the mean and standard deviation values for the pipe weight loss.

Then, using the methods of statistical inference, you can determine the range of difference in weight loss between pipelines built of coated and uncoated pipe. There are two procedures to follow in making this prediction.

- The first is called an "F-test" - this is sort of a "pre-testing" process that lets you know whether or not the second technique - the "confidence interval" - needs any adjustments.
- After the "F-test" is "passed" you then use the "confidence interval" procedure to make your prediction. (Procedures to follow if the "F-test" is *not* passed are examined in the next example).

You'll notice that the "F-test" and "confidence interval" procedures examined here involve mathematical manipulations that will put your advanced professional calculator "through its paces". We'll take it one step at a time.

Keying It In : First, take a statistical look at the Apex Company's sample data, using your calculator to find the mean weight loss, standard deviation, and the square of the standard deviation (used in the tests later) for both coated and uncoated pipe data :

Uncoated Pipe :

Press		Display/Comments	
2nd CA		0	Clear all
2nd FIX	4	0.0000	Fix decimal at 4 places
3.66 Σ+		1.0000	Enter data for
1.28 Σ+		2.0000	uncoated pipe
1.84 Σ+		3.0000	
3.68 Σ+		4.0000	
1.83 Σ+		5.0000	
6.00 Σ+		6.0000	
2nd MEAN		3.0517	Mean weight loss for uncoated pipe
2nd s,σ		1.7653	
x^2		3.1163	

Coated Pipe

Press		Display/Comments
2nd CA		0 Clear all
2nd FIX	4	0.0000 Fix decimal at 4 places
2.66 Σ+		1.0000 Enter data for
0.45 Σ+		2.0000 coated pipe
0.92 Σ+		3.0000
1.69 Σ+		4.0000
0.05 Σ+		5.0000
0.16 Σ+		6.0000
2nd MEAN		0.9917 Mean weight loss for coated pipe
2nd STDEV		1.0213
x²		1.0430

Note · Based on the samples, the mean weight loss for the standard pipe is 3.0517 and the mean weight loss for the Neverust pipe is 0.9917. From these results (without using statistical inference) it appears that the Apex claim of about three times less weight loss for Neverust pipe is justified. But how much can you "depend" on this result ?

Now, for purposes of the "F-test" we will need to identify the data with the greatest standard deviation as the "high" data, and the data with the lowest value standard deviation as the low data. We'll use the subscripts "H" for high and "L" for low to tell these apart · and, in this case, the uncoated pipe data has the greatest standard deviation and will be called the "high" data. Let's tabulate what we have at this point, along with all the necessary labels, below. (The F test here is said to be a "one tailed" test · we're testing to see if Sx_h^2 is greater than Sx_l^2).

Yearly Weight Loss Data (oz/ft/year)	*Uncoated Pipe*	*Coated Pipe*
Sample mean	$3.0517 = \bar{x}_h$	$0.9917 = \bar{x}_l$
Standard deviation	$1.7663 = Sx_h$	$1.0213 = Sx_l$
square of standard deviation	$3.1163 = Sx_h^2$	$1.0430 = Sx_l^2$
number of samples	$6 = n_h$	$6 = n_l$

Now, to conduct the "F-test" you calculate the value of $\frac{Sx_h^2}{Sx_l^2}$, and compare this to an "F-value" found in *Table D* in the *Appendix* (Chapter II-6) The F value you look for in the table should be for $n_h - 1$ degrees of freedom (in this case 6 − 1 or 5) for the numerator and $n_l - 1$ degrees of freedom (in this case 6 − 1045) for the denominator ; and a 95 % degree of certainty

(The F value you will find in this case is F = 5.05). If this F value is *greater* than your calculated value for $\frac{Sx_h^2}{Sx_l^2}$, the F-test is "passed" and you can proceed to the prediction using "confidence interval" procedures. So, to calculate $\frac{Sx_h^2}{Sx_l^2}$.

Press	Display/Comments
2nd CA	0
2nd FIX 4	0.0000
3.1163 ÷ 1.0430 =	2.9878 calculated value of $\frac{Sx_h^2}{Sx_l^2}$

So, in this case, since the F value of 5.05 is greater than your calculated value for $\frac{Sx_h^2}{Sx_l^2}$ of 2.9878, the F test is "passed". Now you can go on and use the "confidence interval" procedure to determine the range of difference in mean weight loss between the coated and uncoated pipe.

To find the range, look in *Table C* in the *Appendix* chapter II-4 and find the t value for the degree of surety you want (here 95 %), and for $n_h + n_l - 2$ degrees of freedom (in this case, 6 + 6 − 2 or 10). (The t value you will find is 2.226) Now you can calculate the *range of predicted difference* for the means, using the fairly complex-looking formula below

Range of difference between means

$$= (\bar{x}_h - \bar{x}_l) \pm \left[\left(\frac{(n_h - 1)Sx_h^2 + (n_l - 1)Sx_l^2)}{(n_h + n_l - 2)} \right) \left(\frac{1}{n_h} + \frac{1}{n_l} \right) \right]^{1/2} t$$

In our case :

$\bar{x}_h = 3.0517$ $n_h = 6$ $Sx_h^2 = 3.1163$ $t = 2.228$

$\bar{x}_l = 0.9917$ $n_l = 6$ $Sx_l^2 = 1.0430$

Press	Display/Comments
2nd CA	0 Clear entire machine
2nd FIX 4	0.0000 Set display to 4 decimal places
3.0517 − 0.9917 =	2.0600 Calculate & store
STO 1	2.0600 $(\bar{x}_h - \bar{x}_l)$

Now calculate the remainder of the formula (AOS is a big help here - just carefully key it in) .

Press	Display/Comments	
( ) ( )		
( ) ( ) 6 [−] 1		
[×] 3.1163 [+]	15.5815	
( ) 6 [−] 1 ( ) [×]		
1.0430 ( ) [+]	20.7965	Value of the numerator
( ) 6 [+] 6 [−] 2 ( )	10.0000	Value of denominator
( ) [×]	2.0797	
( ) 6 [1/x] [+] 6 [1/x] ( )	0.3333	
( )	0.0932	
[√x]	0.3326	
[×] 2.226 [=] [STO] 2	1.8560	Value of right hand expression in formula Now to complete calculation, first add this to $(\bar{x}_h - \bar{x}_l)$
[+] [RCL] 1 [=]	3.9150	Next, subtract second term from $(\bar{x}_h - \bar{x}_l)$
[RCL] 1 [−] [RCL] 2 [=]	0.2050	

Decision Time : So, based on this analysis you can be 95 % sure, based on Apex's data, that the *difference in the means* between a coated and an uncoated pipeline will be between 3.9150 and 0.205 ounces per foot per year. What this means is that you are 95 % certain that the coated pipe process will perform better than the uncoated pipe by as much as 3.9150 ounces per foot per year or by as little as 0.2050 ounces per foot per year (or any value in between). This is all you can tell based on only six experiments. Apex's claim of "up to" three times better performance seems to be technically O.K. - but they left out the "other side" of the claim which could read : "or as little as a few percent better performance ". At any rate all you really have to base your decision on is a predicted range for the *difference between the means*.

With this information it's now time to closely scrutinise what extra costs are involved in changing to the coated pipe, how long the pipeline needs to last, and the other factors surrounding the decision. Your analysis of this data should put you in a better bargaining position with Apex - and also lets you see clearly just how much (or how little) information can be drawn from a small series of experiments.

Corrected "confidence interval" method for analysing change

In this case a young biology student approaches you for help in analysing data he's just taken from an experiment. He is testing to see whether or not a certain drug has any effect on the intelligence level of hamsters - as measured by the time it takes the hamsters to complete a simple "maze" test. Nine hamsters were fed the drug and given the test, while a "control" group of 13, which were *not* treated, were given the same test. The student has already tabulated the data for the two groups of hamsters :

	No Drug	*Treated with Drug*
number of hamsters in sample	13	9
mean time to complete maze	110.02	101.58
standard deviation	9.9116	2.8566
square of standard deviation	98.24	8.16

The student's instructor looked at the data and told the student that it appeared to him that there was no "significant difference" between the two groups. The student, however, feels very sure that the drug *did* create a change. He asks you to determine if he can go back to his instructor and state that he's 99 % sure that the drug really did improve the hamster's performance on the test.

Target : In this case you're trying to determine all you can about the performance of the drug based on a small series of tests. What statistical inference enables you to do is to calculate, at a certainty level you select, a confidence interval (range) of difference in intelligence of hamsters treated with the drug and those not treated with the drug. The method used to calculate this range is a *two part* process. First, an "F-test" is used on the data, then based on the results of this test you calculate either a "corrected" or an "uncorrected" confidence interval.

Tools : To perform the F test we need to identify the data with the greatest standard deviation as the "high" data, and data with the lowest value standard deviation as the "low" data. We'll be using the subscripts "H" and "L" to tell these two groups apart. In this case, we'll tabulate the data we have, with all of the necessary labels, below:

	No Drug	*Treated with Drug*
number of hamsters	$13 = n_h$	$9 = n_l$
mean time on maze test (sec)	$110.02 = \bar{x}_h$	$101.58 = \bar{x}_l$
standard deviation	$9.9116 = Sx_h$	$2.8566 = Sx_l$
square of standard deviation	$98.24 = Sx_h^2$	$8.16 = Sx_l^2$

First, to conduct the F test, calculate the value of $\frac{Sx_h^2}{Sx_l^2}$, and compare this to the appropriate F value found in *Table E* in the *Appendix* chapter II-6 (The appropriate F value in this case is $n_h - 1$ or 12 degrees of freedom for the numerator, $n_l - 1$ or 8 degrees of freedom for the denominator, and a 99 % degree of certainty) At a 99 % degree of certainty, F = 5.67 If your calculated value is less than the value from the table, the F test is "passed" and you can proceed right on to calculating the confidence interval. If, however, your calculated value is greater than the F value you found from the table, a *corrected* "confidence interval" procedure must be used. The F test here is said to be a "one tailed" test, testing to see if Sx_h^2 is greater than Sx_l^2

Keying It In : Begin by calculating $\frac{Sx_h^2}{Sx_l^2}$

Press	Display/Comments	
2nd CA	0	Clear entire machine
2nd FIX 3	0.000	Fix display at 3 decimal places
98.24 ÷ 8.16 =	12.039	Value of $\frac{Sx_h^2}{Sx_l^2}$

Since this value is *greater* than the value found from the F table for this problem (5.67), the F test is *not* passed so a corrected "confidence interval" procedure will be used for the rest of the problem. The object of this calculation is to arrive at a *range for the differences* in performance between hamsters treated with the drug and those not treated with the drug.

The correction to the "confidence interval" procedure essentially boils down to arriving at a *corrected number of degrees of freedom* for the problem. Once this corrected number of degrees of freedom is calculated, then the appropriate t value is used in a formula to calculate the predicted range of difference in the population means.

The corrected number of degrees of freedom is given by the formula

$$\text{corrected degrees of freedom} = \frac{1}{\left[\frac{K^2}{(n_H - 1)} + \frac{(1-K)^2}{(n_L - 1)}\right]} \qquad \text{where } K = \frac{\frac{Sx_H^2}{n_H}}{\left(\frac{Sx_H^2}{n_H} + \frac{Sx_L^2}{n_L}\right)}$$

This is a case where your "advanced professional" machine is a real help in "slicing through" the mathematics. First evaluate K

Press	Display/Comments	
2nd CA	0	Clear entire machine
2nd FIX 3	0.000	Fix decimal at 3 places
98 24 + 13 =		
STO 1 +	7.557	Value of $\frac{Sx_h^2}{n_h}$
(RCL 1 + 8 16		
) + 6 (	8.464	Value of denominator
= STO 2 x²	0.893	Value of K stored in memory 2

Now calculate the "corrected" number of degrees of freedom.

1 ÷	1.000	
(RCL 2 x²) + (		
13 − 1) +	0.066	
(1 − RCL 2)		
x² +	0.011	
(9 − 1) ( ) =	14.734	is the corrected number of degrees of freedom.

Now, to continue with the analysis, this value of the number of degrees of freedom is used to look up a t value from *Table C* in the *Appendix* chapter II-6 (at a 99 % degree of certainty). This t value is used to calculate the range using the formula

Range of difference between means

$$= (\bar{x}_H - \bar{x}_L) \pm \left[\left(\frac{[(n_H - 1)Sx_H^2 + (n_L - 1)Sx_L^2)}{(n_H + n_L - 2)}\right)\left(\frac{1}{n_H} + \frac{1}{n_L}\right)\right]^{1/2} t$$

In our case

$\bar{x}_h$ = 110.02	n_h = 13	Sx_h^2 = 98 24
$\bar{x}_l$ = 101.58	n_l = 9	Sx_l^2 = 8.16

When looking in *Table C* for the approximate value, you'll note that the table only lists t values for integer values of degrees of freedom (14, 15, etc). Using your calculator you can find the appropriate value of t for 14.734 degrees of freedom using a process called "interpolation"

Between degrees of freedom 14 and 16 the t values go from 2.977 to 2.947 To find the t value for 14 734 degrees of freedom :

$2.977 - \left[(14.734 - 14)\ (2.977 - 2.947)\right]$

Press	Display/Comments	
2nd CA	0	Clear entire machine
2nd FIX 3	0.000	Fix decimal at 3 places
2.977 − [ ]	2.977	value of t at 14
[ ] 14.734 − 14 [ ] X	0.734	"distance" from 14 to 14 734
[ ] 2.977 − 2.947 [ ]	0.030	" distance" in t values from 14 to 15
[ ] =	2.955	t value for 14 734

With this t value you can now calculate the range of difference between means using the formula given previously.

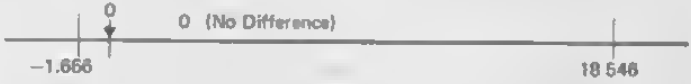
Press	Display/Comments	
2nd CA	0	Clear entire machine
2nd FIX 3	0.000	Set decimal places to 3
110.02 − 101.58 =		
STO 1	8.440	The value of $\bar{x}_h - \bar{x}_l$ stored in memory 1
[ ] [ ]		
[ ] [ ] 13 − 1 [ ] X		Next calculate the
98.24 +	1178.880	righthand term in the
[ ] 9 − 1 [ ] X		equation
8.16 [ ] +	1244.160	
[ ] 13 + 9 − 2		
X	62.208	
[ ] 13 1/x + 9 1/x	0.188	
[ ]	11.697	
√x X 2.955 = STO 2	10.106	
		(Now add $(\bar{x}_h + \bar{x}_l)$
+ RCL 1 =	18.546	Upper limit for difference between means
		Subtract second term from first :

(continued)

(continued)

Press	Display/Comments
RCL 1 − RCL 2 =	−1.666 Lower limit for difference between means

Decision Time : Based on the data you have, you can state with 99 % surety that the difference between the means lies between 18.546 and −1.666. Now if the drug had no effect on the hamsters' performance, you would expect *no difference* (zero difference) between the means. Since the range of predicted values of the difference between means includes the value zero, you *cannot be sure* (at a 99 % degree of certainty) that any real change has taken place when the hamsters are treated with the drug. As the instructor suspected there is no "statistically significant" difference between the two groups at the 99 % confidence level. You might suggest that *more data* be taken at this point.



Going Further . Would this analysis predict a significant difference between the two groups at the 95 % confidence level ?

Answer . Yes.

Keys to Financial Decisions

Healthy money should never lie stagnant, but should be kept in situations where it is "growing" constantly. Sound financial planning should always take this growth into account. There are a variety of business investment and savings situations providing growth for cash. Making plans and predictions in these situations can involve some fairly detailed calculations. Your advanced professional calculator is equipped with several features and functions especially powerful in keeping track of "the value of money".

In this chapter we'll begin with some quite basic cash flow and growth situations, then move on to several more involved examples. Along the way we'll consider personal as well as business decisions involving investments — including a lease or buy decision situation. The formulas and principles involved are not difficult to understand, but the mathematics involved could be really tiresome without your calculator. With your calculator saving time and adding accuracy, planning is easier and faster — and you're free to "try out" many different alternatives without a huge "investment" in time and labour.

One straightforward way to obtain a certain amount of cash growth is to deposit your money in an interest-bearing account. Money in most savings accounts grows according to the "compound interest" formula — which is a basic tool that's important in many financial planning and decision-making situations. We'll consider a simple example and "build up" this formula.

Let's say you deposit £958 in a savings account which pays 0.6 % interest each month. You know you'll be able to leave the money in the account for only three complete months — then you'll need to withdraw most of it for a business purchase. What is the value of the money after three complete months ?

Target : You need to find the value of your deposit (£958) left in a compound interest account for three months, at an interest rate of 0.6 % per month. Along the way we'll develop a formula for calculating compound interest.

Your reasoning might go something like this : At the end of the first month your money had earned 958 x (0.6 %) in interest. So, the total cash accumulated in your account at the end of the first month was 958 + 958(0.6 %). We can rewrite this as 958 x (1 + 0.6 %) — that is, the amount of cash you have at the *end* of the first month is (1 + 0.6 %) times the amount you had at the beginning of the month.

In the same way the amount of money you have at the end of *any* month is just (1 + 0.6 %) times the amount at the beginning of that month. The amount at the end of the second month is :

$[958 \times (1 + 0.6\ \%)] \times (1 + 0.6\ \%)$, which can be written as $958 \times (1 + 0.6\ \%)^2$.

Continuing in this manner, the amount of money you now have at the end of three months can then be expressed as : $958 \times (1 + 0.6\ \%)^3$. This formula is easy to evaluate on your calculator. In addition, the programmable feature allows you to store the calculation sequence in program memory. And by storing the needed data value in memories, you can easily change the amount deposited, the interest rate or the number of periods and repeat the problem as often as you want to.

Keying It In : First decide which data value will be in which memory

Press	Display/Comments
2nd CA	0. Clear everything
958 STO 0	958. Deposit in memory 0.
6 STO 1	0.6 Interest per period in memory 1
3 STO 2	3. Number of periods in memory 2

Now, with the values in memories 0, 1 , and 2, key the problem keystrokes into program memory.

2nd LRN	00 00 Enter learn mode
RCL 0 × (1 +	06 00 Key in problem as written,
RCL 1 %) y^x	11 00 substituting memory recalls
RCL 2 =	14 00 for actual values
2nd	16 00
2nd LRN	0. Exit learn mode

Since the values are already stored in memories, to solve the problem, simply run the program.

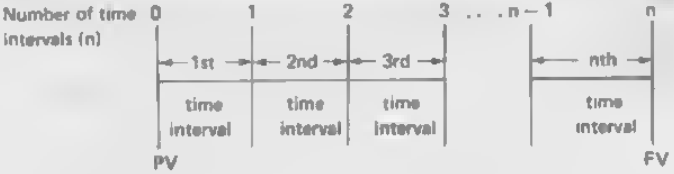
2nd	0. Resets program to step 00
2nd 2	0.00 Set for two decimal display
2nd	975.35 The value of £958 at the end of 3 months.

To find the value after 4 months, just store 4 in memory 2 and run the program

4 STO 2	4.00
2nd	981.20 The value of £958 at the end of 4 months

Going Further : When you are considering more complex problems involving compound interest or payments, a diagram called a *time line* will often help clarify the situation. A time line for this problem would look like this ;

The "value" of your money is shown for the beginning and end of the three months. On time line diagrams there is a notation commonly used to label various points in your problem situation. The amount of money you're starting with now (£958) is called the *present value* (PV) of your money. The value at the end of the three months is called the *future value* (FV) of your money. The 0.6 % interest per month may be labeled i% interest per time interval. The number of time intervals you're considering is usually labeled n, and is marked as shown on the line.



Interest is i% per time interval.

Using these symbols we can now write the "compound interest" formula for calculating future value of money :

$$PV \times (1 + i\%)^n = FV$$

All this formula says is . To calculate the future value (FV) of your money n time intervals from now (at i% interest per interval), just take the present value and multiply it by $(1 + i\%)^n$. There's nothing to it on your calculator.

Here's an important point to keep in mind when using this formula : The interest rate (i%) must be for the *same time interval* used on the time line. We'll be stressing this point as we go along, but keep it in mind — forgetting it is a common source of error in handling these problems.

In many business or everyday life situations you're setting aside a certain amount of cash today for an anticipated *future* purchase or expense. In situations such as this you use the compound interest formula in the "reverse direction". Consider this example :

You're planning to buy a new car in two years for £5000. Your bank has an account available paying 0.8 % per month interest. How much money would you have to deposit *now* to have the £5000 ready then ?

Target : In this case you need to find the *present value* (PV) of a specified future value amount (FV). The time period you're considering is 2 years (24 months) at 0.8 % interest per month.

Tools : In analysing this example you may find a time line helpful.

The basic formula for compound interest we worked up in the previous section is $PV \times (1 + i\%)^n = FV$. In this case you need to calculate a present value, so solve this equation for PV :

$$PV = \frac{FV}{(1 + i\%)^n} \quad \text{or}$$

$$PV = FV \times (1 + i\%)^{-n}$$

This formula just states that if you need to calculate the present value of some future amount, just multiply by $(1 + i\%)^{-n}$.

In our example : $PV = 5000 \times (1 + 0.8\%)^{-24}$.

Press	Display/Comments
2nd [1/x]	0. Clear all
2nd [fix] 2	0.00 Fix decimal at 2 places
5000 × [(] 1 + 8 [%] [)]	
[y^x] 24 [+/−] =	4129.69 = PV

You need to deposit £4,120.69 to have £5000 in two years. Notice the similiarity of this problem to the previous problem. In fact, you can store FV in memory 0, i in memory 1 and n in memory 2, then by adding [+/−] before [=] in the previous program, you can solve for PV with any FV, i, or n value you choose.

PAYROLL DEDUCTIONS
(Future Value of Payments)

Most often in personal or business saving, you're putting away a small amount at regular intervals — building up an account. For handling future value of payments the compound interest formula is just used repeatedly — and as you'll see in this example, a general formula for calculating future value in a regular payment situations can be easily worked up.

Let's say that you've started depositing £75 on the 15th of each month (through payroll deduction) in your company's credit union. You can't get in contact with the credit union right now, but you need to know how much cash you will have accumulated in 6 months, one year, and in 6 years. The credit union pays 0.6 % per month — starting the 1st of the month following the payment.

Target : In this case you went to find the future value that accumulates when you make a series of regular payments for a specified numer of payment intervals

Tools : A time line will be helpful in visualising this example, and we'll work with the compound interest formula

$$FV = PV(1 + i\%)^n \text{ or } PV = FV(1 + 1\%)^{-n}$$

As we go through the solution we'll arrive at a general formula for handling this situation which is easily evaluated on your calculator.

The time line for the 6 payment situation is shown here :

One way to approach the problem is to just calculate the future value of each of the payments at the end of the 6 months using the compound interest formula and then add up all the results. (Often when calculating the future value of an amount of money, it's said that you "move" the money into the future. You may see this term used in discussions on future value.)

So, at the end of the 6th month Pmt_6 will not have earned any interest, Pmt_5 will have earned interest for one month, Pmt_4 will have earned for two months, and so on

The total amount of cash you've accumulated at the end of the 6th month is then

$Pmt_6 + Pmt_5(1 + i\%) + Pmt_4(1 + i\%)^2 + Pmt_3(1 + i\%)^3 + Pmt_2(1 + i\%)^4 + Pmt_1(1 + i\%)^5$.

The programmable feature of the calculator can really be put to work to save you keystrokes in solving this problem. Notice that Pmt (1 + i%) is common for each payment term. Pmt_6 still has (1 + i%) ; however, since it hasn't had time to gain interest $Pmt_6 \times (i + i\%)^0 = Pmt_6 \times 1$, or simply Pmt_6. Since the payments are equal, the only difference between terms in our equation is the number of periods, n, which increments from 0 to 5. The following key sequence shows how to key in only one payment term and let the calculator do the repetitive work. The value for n is in memory 0 and you simply add 1 to memory 0 for each subsequent calculation. The subsequent calculations are summed together by leaving a [+] pending at the end of each calculation.

Press	Display/Comments
[2nd] [CA] [2nd] [lrn]	00 00 Clear all and enter learn mode
75 [×] [(] 1 [+] .5 [%] [)]	10 00 Key in payment term.
[y^x] [RCL] 0 [×] 1 [SUM] 0 [+]	18 00 Sum 1 to memory 0 and leave + pending
[2nd] [R/S] [2nd] [RST]	20 00 Stop to show result, get ready to reset.
[2nd] [LRN]	0 Exit learn mode.

Now run the program as follows .

[2nd] [RST]	0. Resets program to step 00.
[2nd] [FIX] 2	0.00 Fix decimal at 2 places.
[2nd] [R/S]	75.00 One month
[2nd] [R/S]	150.38 Two months
[2nd] [R/S]	226.13 Three months
[2nd] [R/S] [2nd] [R/S] [2nd] [R/S]	456.66 Six months

You will have £455.66 at the end of six months You can continue for as many months as you wish. To start over, just press [CLR] [STO] 0 then [2nd] [R/S] again

Going Further : There's an easier way – a real time saver for problems like this. A series such as the one we've just worked with $1 + (1 + i\%) + (1 + i\%)^2 + (1 + i\%)^3 + (1 + i\%)^4 + (1 + i\%)^5$ is called a geometric series of 6 terms. The sum of a series like this can be simplified to .

$$\frac{((1 + i\%)^6 - 1)}{i\%}$$

So the amount you have at the end of the 6 months can be written as :

$$75 \times \left(\frac{((1 + 0.5\%)^6 - 1)}{0.5\%} \right)$$

Now you need to key the new problem into program memory. Notice that this problem does not require summation of successive calculation, and again the value n is assumed to be stored in memory 0 to make it convenient to repeat the calculation for a different number of periods.

Press	Display/Comments
2nd CA 2nd LRN	00 00 Clear all and enter learn mode
75 X (1 + 5 %)	12 00 Key in formula as written.
y^x RCL 0 − 1) + .5	22 00
%) = 2nd R/S 2nd RST	26 00
2nd LRN	0 Exit learn mode.

Now run the program.

2nd Fix 2nd Ix 2	0.00 Reset and fix decimal.
5 STO 0 2nd R/S	465.66 Amount after six months

Notice that this is the same result found before by the "long" method. Do not clear all or turn off calculator

Now in general, the future value (FV) will be

$$FV = Pmt \times \left(\frac{((1 + I\%)^n - 1)}{I\%} \right)$$

Using this general formula it's now easy to compute the amount you will have in your savings account at the end of one year :

$$\text{Value at 12 months} = 75 \times \left(\frac{((1 + 0.5\,\%)^{12} - 1)}{0.5\,\%} \right)$$

Press	Display/Comments
12 STO 0 2nd	925.17 At the end of one year

The amount you will have at the end of 5 years is :

$$\text{Value at 60 months} = 75 \times \left(\frac{((1 + 0.5\,\%)^{60} - 1)}{0.5\,\%} \right)$$

Press	Display/Comments
60 STO 0 2nd	5232.75 At the end of five years

Decision Time : Using this formula and your calculator it's now easy to predict and check out "what if" for any set of payroll deductions at various interest rates. Such calculations enable you to quite easily and accurately examine various investment and savings alternatives.

YOU'RE RICH – BUT . . .

11-5

(Present Value of a series of payments)

Your Uncle Clyde liked you pretty well, but he didn't have much faith in your money handling ability. That's probably why he left your brother the farm and you with a series of payments. The will said that you were to receive £1000 per year for the next 8 years (in one lump sum at the end of the year). An urgent business "opportunity" has arisen – and you'd like to have as much cash as possible right now.

Making a quick call to the bank you find that they'll gladly buy the series of payments from you at their present value – figured at 8 % annual interest. Your problem – is this a wise decision ?

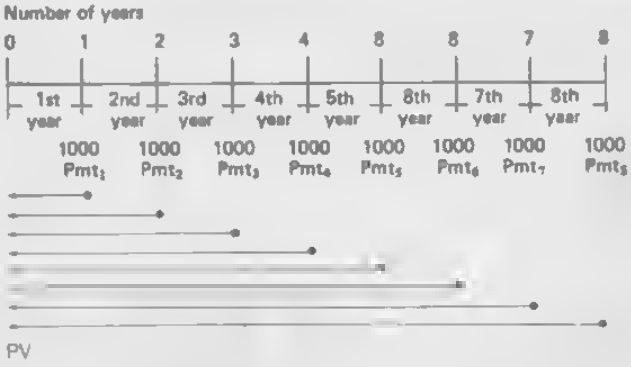
Target : You need to find the present value of the payments (how much the money is worth now).

Tools : A time line will help you visualise the situation, and we'll also work with the formula to calculate the present value of some future amount .

$PV = FV\ (1 + i\%)^{-n}$

As we go through the solution we'll arrive at a general formula for handling this situation which is easily evaluated on your calculator.

The Time Line .



One way to approach the problem is to just calculate the present value of each of the payments and then add up all of the results

$PV = Pmt_1 (1 + i\%)^{-1} + Pmt_2 (1 + i\%)^{-2} + Pmt_3 (1 + i\%)^{-3} + Pmt_4 (1 + i\%)^{-4} + Pmt_5 (1 + i\%)^{-5} + Pmt_6 (1 + i\%)^{-6} + Pmt_7 (1 + i\%)^{-7} + Pmt_8 (1 + i\%)^{-8}.$

Now since all of the payments are equal you "factor them out" to get :

$PV = Pmt [(1 + i\%)^{-1} + (1 + i\%)^{-2} + \ldots (1 + i\%)^{-8}]$

There's an easier way. The series :

$(1 + i\%)^{-1} + (1 + i\%)^{-2} + \ldots (1 + i\%)^{-6}$

is another geometric series of 8 terms. The sum of a series like this can be simplified to .

$$\left(\frac{(1 - (1 + i\%)^{-8})}{i\%}\right)$$

Now, in general, for any series of payments of equal amounts at the *end* of n equal time intervals, the present value will be

$$PV = Pmt \frac{(1 - (1 + i\%)^{-n})}{i\%}$$

Now let's find the present value of the £ 1000 a year payments. In our case,

$$PV = 1000 \times \left(\frac{(1 - (1 + 8\%)^{-8})}{8\%}\right)$$

Keying It In :

Press	Display/Comments
2nd CA	0. Clear all
2nd % 2	0.00 Set decimal to 2 places
1000 × (	1000.00 The amount of yearly payment
1 − (1 + 8 %)	1.08 The (1 + 8 %) term
y^x 8 +/−) +	
8 %) =	5746.64 PV

Decision Time : Now's the time to closely examine your "business opportunity". Decide what the prospects of success are very carefully – before deciding to accept £5746 64 of ready cash

Going Further : You were dissatisfied with the present value of the payments so you had your uncle's lawyer recheck the will. He found the provisions read that you may receive the payments at the *first* of the year. You'd like to compare the present value of the payments if they are made at the first of the year with the present value of the payments made at the end of the year. One way to approach the problem is to calculate the present value of each of the paymants, using the present value formula .

$$PV = FV\,(1 + i\%)^{-n}$$

The present value of the payment is $Pmt_1 + Pmt_2\,(1 + i\%)^{-1} + Pm_3\,(1 + i\%)^{-2} + Pmt_4\,(1 + i\%)^{-3} + Pmt_5\,(1 + i\%)^{-4} + Pmt_6\,(1 + i\%)^{-5} + Pmt_7\,(1 + i\%)^{-6} + Pmt_8\,(1 + i\%)^{-7}$.

Notice Pmt_1 is already in present value. Since all the payments are equal, you can "factor them out" to get

$$Pmt \times [1 + (1 + i\%)^{-1} + (1 + i\%)^{-2} + \ldots (1 + i\%)^{-7}]$$

Again, there's an easier way. The series

$1 + (1 + i\%)^{-1} + (1 + i\%)^{-2} + \ldots (1 + i\%)^{-7}$ is another geometric series. The sum of a series like this can be simplified to

$$\left(1 + \frac{(1 - (1 + i\%)^{-(7)}}{i\%}\right)$$

Now, in general, for any series of payments of equal amounts at the *beginning* of n equal time intervals (in advance) earning interest of i% per time interval, the present value will be

$$PV = Pmt\left(1 + \frac{(1 - (1 + i\%)^{-(n-1)}}{i\%}\right)$$

Now let's find the present value of the £1000 a year payments made at the beginning of each year for 8 years at 8 % annual interest.

$$PV = 1000 \times \left(1 + \frac{(1 - (1 + 8\%)^{-(8-1)})}{8\%}\right)$$

Press	Display/Comments
2nd CA	0. Clear all
2nd Fix 2	0.00 Set decimal at 2 places
1000 × (1 + (1 − (1 + 8 %) y^x 7) ÷ 8 %) ×	6206.37 The present value of the payments if they're made at the beginning of each year

Let's find how much more the present value of the payments is if payments are made "in advance" (at the beginning of each period) or "in arrears" (at the end of each period).

5746.64 469.73

Decision Time : Now you may want to re-evaluate your decision based upon the increase in present value due to receiving the payments in advance.

YOU'RE THE BANKER

(Calculating loan payments)

11-5

Business has been going pretty well for you lately and the word is getting around. In fact, your brother-in-law has asked you for a loan. He wants to borrow £4635 to improve his home. He's willing to pay you 6 % annual interest, and claims he'll pay the loan off in 3 equal yearly payments starting next year. You decide to help him out, and need to calculate just how much the loan payments should be.

Target : You need to determine the amount of the loan payments for your brother-in-law. Along the way we'll work up a general formula for calculating loan payments.

Tools : We'll be using a time line to help visualise the problem, along with the formula for present value of a series of payments received at the *end* of each payment period. (When payments are received at the *end* of the periods you're considering, they're said to be received "*in arrears*". Payments received at the beginning of the period are said to be received "in advance".)

$$PV = Pmt\left(\frac{(1-(1+i)^{-n})}{i}\right)$$

The Time Line

The present value of the payments from your brother-in-law for 3 years at 6 % is £4535 and using the above formula you can write :

$$4535 = Pmt \times \left(\frac{(1-(1+6\%)^{-3})}{6\%}\right)$$

Solving for the Pmt gives

$$Pmt = \frac{4535}{\left(\frac{(1-(1+6\%)^{-3})}{6\%}\right)}$$

Keying It In :

Press	Display/Comments
2nd CA	0. Clear all
2nd FIX 2	0.00 Fix decimal at 2 places
4536 + ((1 . − (1	
+ 6 % ((y^x 3 +/−	
) + 6 %) =	1696.59 Pmt

Decision Time : The annual payments should be £1696.59. Do you think your brother-in-law will come up with the money ?

MOW YOUR OWN LAWN ?
(Buy equipment or contract for service ?)

The old lawnmower "bit the dust", and you ask yourself if you should buy a new lawnmower at all. You don't particularly like mowing the lawn, trimming the hedge, pulling weeds, etc . . but it will cost £200 per year to have someone contract to take care of the garden

The price of a new lawnmower is £329,95 and if you were to buy it you would expect to pay about £8 per year for operating and maintenance expenses. The mower would probably last about 6 years, and then you would get rid of it (donate it to a garage sale to help raise money for the school band or some such cause) Keeping in mind that your cash would grow at about 7 % per year if you kept it in the bank, you're wondering if you really wouldn't be better off just having the garden done.

Target : You want to take a sound, "cold" look at the situation What's the *real* difference in cost between paying to have the job done, and buying a new lawnmower to do it yourself ?

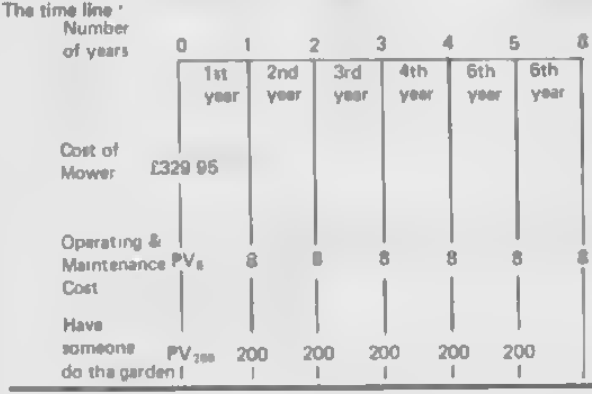
Tools . A time line will again be used to help in picturing what's going on We'll also be using the formula for the present value of a series of payments "in arrears"

$$PV = Pmt\left(\frac{(1-(1+i\%)^{-n})}{i\%}\right)$$

and also the formula for the present value of a series of payments in advance :

$$PV = Pmt\left(1+\frac{(1-(1+i\%)^{-(n-1)})}{i\%}\right)$$

The time line :



One way to approach this problem is to consider the fact that if you didn't buy the mower the £329.95 cost and £8/year operating cost could be earning 7 % annual interest. So you would find the present value of these amounts (labeled $PV_{329.95}$ and PV_8 on the time line).

$$PV_{329.95} = 329.95$$

$$PV_8 = 8 \times \left(\frac{(1 - (1 + 7\%)^{-6})}{7\%} \right)$$

Now you want to compare this to the cost of having the lawn done (PV_{200}). In situations like this, use the formula for the present value of a series of payments in advance :

$$PV_{200} = 200 \times \left(1 + \frac{(1 - (1 + 7\%)^{-(6-1)})}{7\%} \right)$$

Keying It in :

Note the similarity of the two problems. Keystrokes can be saved by entering the "similar part" of the calculation into program memory. Memory 0 is used to store n or n−1 values.

Press	Display/Comments
2nd CA 2nd LRN	00 00 Clear all and enter learn mode.
(1 − (1 + 7 %)	09 00
y^x RCL 0 +/−) ÷ 7 %	17 00 Partial equation $\frac{(1 - (1 + 7\%)^{-(n)}}{7\%}$
= 2nd R/S 2nd RST	20 00
2nd LRN	0 Exit learn mode

Now solve for PV_s

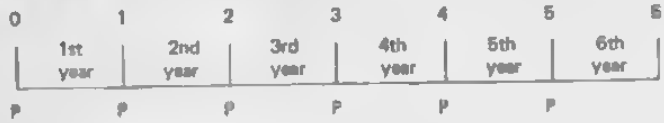
6 STO 0 2nd FIX 2	6.00 Store n in memory 0.
2nd π 8 × (2nd R/S	38.13 = PV_8
+ 329.95 =	368.08 This is the present value of the mower and the maintenance payments.

Now compare this to the present value of the cost of having the lawn done

Press	Display/Comments
5 STO 0	5.00 Store $n-1$ in memory 0.
200 × (1 + 2nd n!	1020.04 $= PV_{200}$
− 368.08 =	651.96 The difference

Decision Time : The difference in cost between buying the mower and mowing it yourself and paying someone else to mow it is £651.96. The decision – can you afford it ?

Going Further : While your calculator is still turned on, maybe you would like to know how much you can pay yourself at the first of each year to mow your own lawn.



651.96

To find how much each of these payments will be use the formula for the present value of a series of payments in advance :

$$651.96 = Pmt\left(1 + \frac{(1-(1+7\%)^{-(6-1)})}{7\%}\right)$$

Solving for the Pmt gives :

$$Pmt = \frac{651.96}{\left(1 + \frac{(1-(1+7\%)^{-(6-1)})}{7\%}\right)}$$

Keying It In :

Press	Display/Comments
	651.66 Previous result
+ (1 + 2nd RCL	127.83

You can pay yourself £127.83 each year to keep your own lawn At this point you can look at your decision in a new light It is worth it ?

BUY A RENTED HOUSE ?

(Investment decision)

11-5

You're considering buying a house that is presently rented for £375 per month as an investment. You have £10,000 available cash for the investment. You realise that buying a house involves some risk, so you are planning the move only if you can make a sizeable profit on the deal (15% annual rate).

After checking with a broker, you find that you can buy the house by placing £10,000 down and assuming a £25,000 mortgage. You figure that your expenses, including mortgage payments, will be about £260 per month. You expect to keep the property for 10 years, sell the property, pay off the mortgage, and net £20,000. Should you invest in the house ?

Target : Analyse the situation and see if you can achieve your overall goal of 15 % effective annual interest on your investment.

A Note on Interest : You can calculate equivalent interest rates with different compounding intervals by using the compound interest formula. For example, suppose you deposit amount (A) for a year at 15 % per year interest.

(You would have A (1 + 15 %) at the end of the year.) Now say you would like to know what monthly interest rate (i%) is equivalent to 15 % per year. To determine the equivalent monthly rate, assume you deposit the same amount (A) in an account which pays i% per month.

The interest rates will be equivalent if the final amounts are equal

$A(1 + 15\%) = A(1 + i\%)^{12}$

Now the equation may be solved for i%.

Dividing both sides by A gives :

$(1 + 15\%) = (1 + i\%)^{12}$

taking the 12th root, $\sqrt[12]{(1 + 15\%)} = (1 + i\%)$

subtracting 1 from both sides,

$\sqrt[12]{(1 + 15\%)} - 1 = i\%$

On your calculator [2nd] [CA] [(] 1 [+] 15 [%] [)] [x^y] 12 [−] 1 [=] :
0.0117149 or about 1.17 % per month interest.

Tools : The basic tools in analysing these situations are a time line diagram, and the compound interest formula :

$$PV = FV\,(1 + i\%)^{-n}$$

as well as the formula for the present value of a series of payments in advance .

$$PV = Pmt\left(1 + \frac{(1 - (1 + i\%)^{-(n-1)})}{i\%}\right)$$

The Time Line . The interest is 1.17 % per month (and 10 years is 120 months).

You will be receiving £375 each month for rent, and spending £250 each month for the mortgage payment and maintenance. You can consider that the net result is actually a series of monthly payments of £125 in your favour, and consider them on the same time line

Also, at the end of the 10 years you will receive £20,000 cash from the sale of the house.

One way to arrive at a conclusion on this investment is to compare your £10,000 investment to the £125 payments plus the £20,000 selling price (assuming that you make the 1,17 % monthly interest desired) If the present value of the net payments plus the present value of the selling price exceed your £10,000 investment amount — the investment is a sound one.

Step (1) .Find the present value of the £125 monthly income Note that the payments are made at the beginning of each month , therefore use the formula for the present value of a series of payments in advance

$$PV = 125 \times \left(1 + \frac{\{1 - (1 + 1.17\ \%)^{-(120-1)}\}}{1.17\ \%}\right)$$

Step (2) . Find the present value of the £20.000 you expect to make when you sell the house .

$20{,}000 \times (1 + 1{,}17\ \%)^{-120}$

Now add the results from step (1) and step (2) to get the present value of the income from the investment.

Keying It In :

Press	Display/Comments
2nd CA	0. Clear all
2nd fix 2	0.00 Set display at 2 decimal places
125 × (1 +	Calculate Step (1)
(1 − (1 +	
1.17 %) y^x	
119 +/−)	
+ 1.17 %) =	8132.27 The present value of the payments if you make 1 17 % monthly
	Calculate Step (2)
20000 × (1 +	
1.17 %) y^x 120	
+/− =	4952.45 The present value of the 20,000 selling profit. Now add the two present values together.
+ 8132.27 =	13084.72

Decision Time : Based on this analysis, the investment is a good move. The income from the house will actually be creating revenues for you that are equivalent to a 1.17 % per month return or 15 % per year on an initial investment of £13,084 72. Since you can "purchase" these profits for only £10,000, the house is indeed a sound investment.

LEASE IT UP . . .

(Lease or buy decision)

II-5

Your business is considering buying or leasing a new computer. According to the financial lease agreement, the company would pay £36,000 par year for 5 years. The company could buy the machine outright (including a 5 year service contract) for £135,000. You have determined that no resale value on the machine can be expected after 5 years. If the computer is installed, it is expected to save the company £46,000 per year. Your company expects a yearly return of 15 % on all funds of this sort invested. The company is a healthy one, with good credit, and can borrow at 8 % annual interest from local banks.

Target : In this case you want to analyse the situation in two parts
Decision 1 : Is it cheaper to lease or buy the computer, based on the data you have and the company's financial situation ? Once you've made that decision, then you want to go on to
Decision 2 : Is it a sound investment for the company to acquire the computer at all, based on the 15 % yearly return the company requires on investments ?

Tools : Again a time line diagram will be used to get a picture of the situation, and the cash values you'll need to make your decisions will be calculated using the formula for the present value of a series of payments "in arrears" (received at the end of the period) :

$$PV = Pmt\left(\frac{(1-(1+I\%)^{-n})}{I\%}\right)$$

and the present value of a series of payments in advance formula :

$$PV = Pmt\left(1+\frac{(1-(1+I\%)^{-(n-1)})}{I\%}\right)$$

Decision 1 : Should you lease or buy the computer ? The loan rate is 8 %.

Find the present value of the payments if the computer is leased, which is also called the Equivalent Purchase Price (EPP) of the lease.

$$EPP = 36000\left(+\frac{(1-(1+8\%)^{-(8-1)})}{8\%}\right)$$

Keying It In :

Press	Display/Comments
2nd CA	0. Clear all
2nd fix 2	0.00 Set decimal at 2 places
36000 × (1 + I 1) − (1 + 8 %) y^x 4 +/−) + 8 %) =	155236.57 The Equivalent Purchase Price of the lease.

Decision Time : If the money is borrowed at 8 %, the Equivalent Purchase Price of the lease is £155 236.57. Comparing this to the purchase price of £135.000, purchasing the computer is the better alternative.

Decision 2 . Should you acquire the computer at all ? The critical question is : Is the present value of the savings the computer generates — assuming a 15 % return — enough to justify the £135.000 investment ?

	0	1	2	3	4	5
		1st year	2nd year	3rd year	4th year	5th year
Investment compared to Payments	PV_2	46 000	46 000	46 000	46.000	46 000

Find the present value of the £46,000 yearly savings, PV_2 .

$$PV_2 = 46000 \times \left(\frac{(1 - (1 + 15\,\%)^{-5})}{15\,\%} \right)$$

Then, compare the present value of the savings to the £136 000 investment

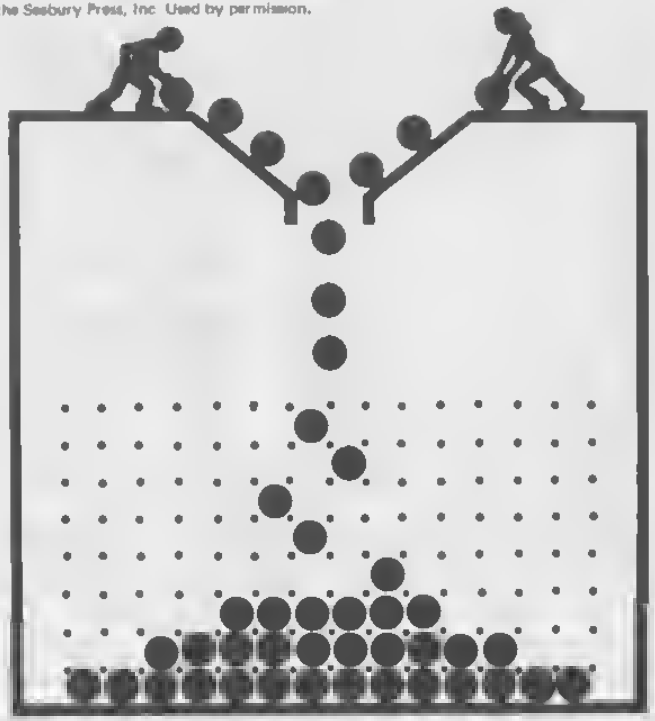
Keying It In :

Press	Display/Comments
2nd CA	0. Clear all
2nd fix 2	0.00 Set decimal at 2 places
46000 × ((1 −	
(1 + 15 %) y^x 5	
+/−) + 15 %)	
=	**164199.13** The present value of the savings.

Decision Time : Acquiring the computer is a good move ! The savings it creates amount to an equivalent of a 15 % return on an investment of £164,199.13. You are achieving this level of return with an actual investment of only £136,000 – so the investment would be a sound one – based on this analysis

A Little Theory...

"The mathematical order of the universe is our answer to the pyramids of chaos. On every side of us we see bits of life that are completely beyond our understanding - we label them unusual, but we really don't want to acknowledge them The only thing that really exists is statistics. The intelligent person is the statistical person".



As mentioned in the introduction to this book, our primary concentration is on the "how to use" side of the statistical tools that are especially applicable in a variety of business and everyday life situations. We hope you find these techniques valuable (and even enjoyable) to use in bringing more accuracy into your decision making with your calculator. But - there are always those folks who ask Why and how does this stuff work, anyway ?

The full answer to all parts of that question probably would involve an extended statistics course - and some sources for further reading are suggested in the *Bibliography.* For those of you who'd like to brave a quick survey of the key elements of what we've been using here we go .

Start with a Simple Example (Small Population)

To get into the theory let's start with an analysis of a small body of data - one we can completely handle Suppose you are examining test scores for 5 people on a simple exam (statisticans would say that this *population* consisted of *5 elements*) Let's say that (out of a perfect score of 10) the scores for the 5 pupils are 4, 5, 8, 7 and 8 With your calculator you can immediately and easily calculate the *population mean* (labeled μ) and the population *standard deviation* (labeled σ) :

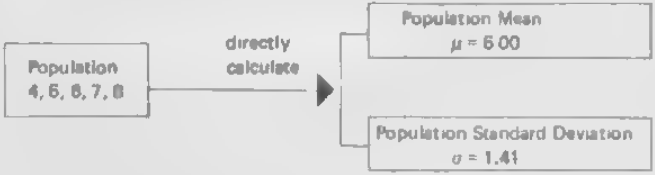
Press	Display/Comments	
2nd CA	0	Clear entire machine
2nd FIX 2	0.00	Set display to readout 2 decimal places
4 $\Sigma+$	1.00	
5 $\Sigma+$	2 00	Calculator keeps a count
8 $\Sigma+$	3.00	of your data entries
7 $\Sigma+$	4 00	
8 $\Sigma+$	5 00	
Now to find the population mean (μ)		
2nd MEAN	6 00	

To find the *population standard deviation* (σ) you use the special key sequence :

2nd VAR $\sqrt{x}$	1.41	

(We'll be saying more about why you use *this* sequence to find σ a little later).

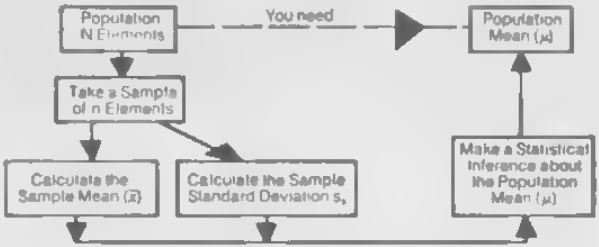
So with this *small population* you can easily and directly analyse all the data in straightforward fashion as illustrated below



What About a Huge Population ?

Now here's the rub - what happens when you'd like to know what the population mean (μ) really is, but the population is made up of thousands (or even millions) of items ? Even with your calculator helping, entering all that data may be nearly impossible. In addition, sometimes the measurement you're taking may *destroy* the item. For example - imagine that you're testing a shipment (population) of 5000 batteries to check on their lifetime. You have to deplete a battery to know how long it lasted in the lifetime test. If you did this to the entire population - you'd know *exactly* what the mean lifetime for the population was. You'd also have no batteries left to use !

In situations like this, one logical alternative is to select a smaller number of items from the population - a *sample* - and test them. This is where the science of statistics comes in. Based on analysing the *smaller* sample - which is cheaper, easier (and more possible) than testing the population - you can use *methods of statistical inference* to make statements about the population mean (μ). Your first step would be to calculate the sample mean ($\bar{x}$) and the sample standard deviation (s_x). Then you'd apply some statistical techniques to get back to information about the population, as diagrammed below



Now - the "roundabout" path by which you use your sample data to get back to information about the population contains *some* chance. It's fairly logical that the *larger* your sample *is,* the less chancey your statements about the mean are

The Sample and the Population

Now we focus on one key point in the situation. When you take a small sample out of the large population - how "representative" is it ? In our battery example if you just happened to pick a lot of "duds", you could wind up underrating an entire shipment based on their performance. Likewise, if by chance you only picked real "winners" - you may overrate the shipment. In this discussion we'll be focusing on what the chances are that the sample mean ($\bar{x}$) is near the population mean (μ).

To give you a feel for how statisticians study this situation - we'll go back to our small *population* of 5 tests scores (one we can handle) and consider the situation when *samples* of 2 test scores are taken from it and examined. (In practice a population this small would not be handled using statistics - but by using it to examine the *processes* statisticians use - we'll demonstrate some important concepts). Now concentrate on this process

As we've already discussed, our five test scores were 4, 5, 6, 7, 8. The population mean (μ) was 6. What would happen if you picked out 2 of these scores at random (a sample) - and checked *their* mean ($\bar{x}$) ? What would your chances be that the *sample mean* would also be 6 - equal to the population mean ?

To answer this timely and interesting question you first need to focus on *all the possible samples of two test scores* you can draw out of our population of 5, and then examine the sample means for each possibility. This is the way statisticians first began looking at the problem of statistical inference. We'll tabulate all of the possible samples of 2 test scores, along with their means, in the table on the following page.

(Note that the method of selection for the test scores at random could be visualised as - putting each score on a slip of paper - putting the papers into a hat, shaking well, picking one out and noting it - *replacing it in the hat* - shaking again - picking again. The replacement factor is important. When samples are taken without replacement from a population of N elements, a "correction factor" is entered into statistical inferences).

Population of 5 Test Scores : *4, 5, 6, 7, 8*

In this table we are tabulating all possible ways of picking a sample of 2 elements as well as the mean of each sample.

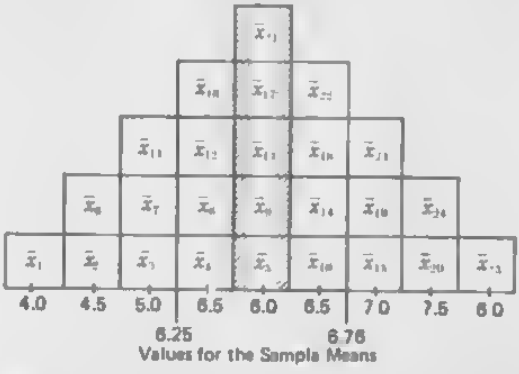
All Possible Samples of 2 Elements	Value of the Mean for Each Sample	Label for Mean Value
4 4	4 0	$\bar{x}_1$
4 5	4 5	$\bar{x}_2$
4 6	5 0	$\bar{x}_3$
4 7	5 5	$\bar{x}_4$
4 8	6 0	$\bar{x}_5$
5 4	4 5	$\bar{x}_6$
5 5	5 0	$\bar{x}_7$
5 6	5 5	$\bar{x}_8$
5 7	6 0	$\bar{x}_9$
5 8	6 5	$\bar{x}_{10}$
6 4	5 0	$\bar{x}_{11}$
6 5	5 5	$\bar{x}_{12}$
6 6	6 0	$\bar{x}_{13}$
6 7	6 5	$\bar{x}_{14}$
6 6	7 0	$\bar{x}_{15}$
7 4	5 5	$\bar{x}_{16}$
7 5	6 0	$\bar{x}_{17}$
7 6	6 5	$\bar{x}_{18}$
7 7	7 0	$\bar{x}_{19}$
7 6	7 5	$\bar{x}_{20}$
8 4	6 0	$\bar{x}_{21}$
8 5	6 5	$\bar{x}_{22}$
8 6	7 0	$\bar{x}_{23}$
8 7	7 5	$\bar{x}_{24}$
8 8	8 0	$\bar{x}_{25}$

Now, in a "real life" situation you'd be picking out a sample, measuring *its* mean value ($\bar{x}$), and from that result, trying to calculate or deduce the population mean value (μ). So, *focus your attention on the sample mean values.* This is the data that's available to you - you're actually picking out sample mean values, $\bar{x}$'s, and need to know what your chances are that $\bar{x}$ comes close to the actual value of the

population mean (μ). (Remember that your population mean here is 6 - glance at the table and get a "feel" for your chances of picking an $\bar{x}$ of 6 at random).

What Are Your Chances ?

Let's get a picture of how the sample means (the $\bar{x}$'s) vary. We can do this in a simple picture that puts each mean value in its place as shown below



Values for the Sample Means

In this picture we've just put each mean label inside a little "box", and stacked up the boxes according to the value of their means. This picture gives you a feel for what your chances would be of picking a sample at random, and finding one with a sample mean equal to the population mean value of 6. Five of the sample means $\bar{x}_{21}$, $\bar{x}_{17}$, $\bar{x}_{13}$, $\bar{x}_9$ and $\bar{x}_5$ (in the centre boxes) each have mean values of 6. In fact, the most probable choice would be a 6 value. It turns out that for large populations (N over 100) and large samples (n over 30), this is a general rule :

The most probable value of $\bar{x}$ is the population mean (μ).

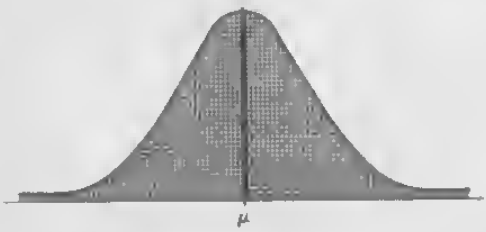
Relative Areas

If you examine the *relative areas* of the boxes you can get a visual picture of the chances that if you pick a sample at random - its $\bar{x}$ will be a 6. There are 26 boxes in all, 6 of them contain 6's, so your chances actually can be visualised as the ratio of the shaded boxes to the total area of all the boxes .

or $\frac{5}{25}$ or 20 %.

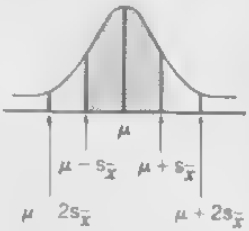
In this pictura you can also consider the values of $\bar{x}$ that are *close* to μ. What would your chances be of picking a sample at random whose x was 6 ± 0.75 ? (An $\bar{x}$ value from 5.25 to 6.75 ?) Again - just count all the boxes containing x's between 5.25 and 6.75, and divide by the total number of boxes. Your chances in this case would be $\frac{13}{25}$ or 52 %.

Now - let's move from our simple little population and consider for a moment what would happen to our picture if the number of elements in the population increased from 5 to 100, and the sample size were increased from 2 to 30. If we were able to take all the sample means and arrange them pictorially as we did before - we'd see something like the behaviour below .



As the boxes get smaller and smaller (they'd be very small in this case, for N = 100 and n = 30 there are 10^{40} boxes) - the outside of our pictura smooths out into a classic, symmetric, very important shape - called the *Normal Curve*. As a general rule of thumb, it's assumed in most situation that the $\bar{x}$'s are *distributed normally* (follow the normal curve) whenever the population has over 100 elements and the sample size is greater then 30.

Much has been written about the normal (or "Bell" or "Gaussian") curve - but we'll be focusing on the *areas under the* curve, and how we can use the curve to get more information about the population mean from the sample mean. We'll now be introducing the role of another key "player" - the *standard deviation of the sample means*, which is labeled $s_{\bar{x}}$.



It turns out that because of the fact that the sample means follow this "normal" behaviour" (for large populations and samples) some mathematical predictions can be made using the normal curve that apply to just about any situation where large populations and samples are concerned. We'll present some of these useful *results* here. Remember that statisticians calculated these results by examining areas under the normal curve.

First of all, examine the normal curve above and note that we've partitioned its area into 4 sections, each of which is separated by $s_{\bar{x}}$. Look at the shaded area - it includes all the sample means whose values are between $\mu - s_{\bar{x}}$ and $\mu + s_{\bar{x}}$. It turns out that ratio of this area to the total is always the same – 68.26 %. This means that whenever you pick a sample from a population, your chances are 68.26 out of 100 that you have picked a sample mean that's within $\pm s_{\bar{x}}$ of the population mean. Another way of saying this is that you can be 68.26 % sure that the *population mean* lies somewhere in the range of *your sample mean* plus or minus $s_{\bar{x}}$

Now it turns out that $s_{\bar{x}}$, the *standard deviation of the sample means*, is fairly easy to calculate from your sample data. For samples with larger than 30 elements ($n > 30$), $s_{\bar{x}}$ can be considered equal to $\frac{s_x}{\sqrt{n}}$, where s_x is just the *sample standard deviation*. The sample standard deviation is readily available - this is the number you see displayed in your calculator after you enter your sample data (with the [Σ+] key), and then press [2nd] [STDEV].

Recapping - Getting to the Predicted Range for μ

So now with the help of the normal curve you can analyse a population, based on a sample, in the following way :

First, find the sample mean ($\bar{x}$) and sample standard deviation ($s_{\bar{x}}$), by entering the sample data into your calculator with the [Σ+] key, and then using the [2nd] [MEAN] and [2nd] [STDEV] key sequences.

Then, using your sample data you can say with 68.26 % certainty that the population mean (μ) lies between $\bar{x} + \frac{s_{\bar{x}}}{\sqrt{n}}$ and $x - \frac{s_{\bar{x}}}{\sqrt{n}}$

That is, you can use your sample data to set up a *predicted range for the population mean*. This range is as close as you can get to the population mean - because of the uncertainty in the process of using *sample* data to draw conclusions about the *population*. You can only state, to a certain degree of certainty, that the population mean lies somewhere in that range.

Analysing with Large Samples: z Scores

Notice that the predicted *range* for the population mean above, gave us the limits for the value of μ to one specific degree of certainty . 68.26 %. In most applications it's advisable for *you to be able to select the degree of certainty* that you desire (or *need*) when making any decision about a population, based on sample data

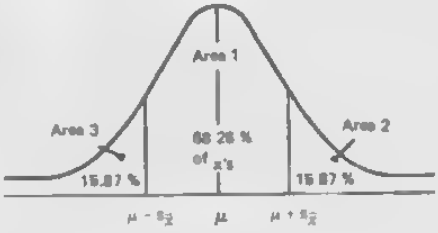
To make this easy to do, tables have been constructed based on the areas under different portions of the normal curve. These tables are called tables of "z values" or "z scores", and they enable you to calculate a predicted range for μ to a degree of certainty you select. (A "z value" table is included for your use in the Appendix).

To use the table- just decide *how sure you want to be* that your calculated range will include the population mean. Check in the z table to find the appropriate z score.

Upper/Lower Limits

Note that two columns are included in the z table. The column you use to find your z score depends on your particular decision situation - as you'll see in the examples in various chapters of this book. If your decision involves *just* an upper or lower value for μ - just one ' limit" - use column I, otherwise, use column II.

To explain why the z values are different for these two situations, consider the normal curve that we have been discussing.



Your chance of picking an $\bar{x}$ in area (range $\mu \pm s_{\bar{x}}$) is 68.26 % as discussed earlier. Looking at this another way, your chance of picking an $\bar{x}$ outside of area 1 is

$\frac{\text{area 2 + area 3}}{\text{total area}}$ or $\frac{17.67\ \% + 17.87\ \%}{100\ \%}$

or about $\frac{38}{100}$. But suppose you are *only* interested in your chance of picking an $\bar{x}$ greater than $\mu + s_{\bar{x}}$ (checking only an upper limit). Your chance is $\frac{\text{area 2}}{\text{total area}}$ or $\frac{17.87\ \%}{100\ \%}$ or about $\frac{18}{100}$. Since different proportions of the total area are used, different z scores must be used for these two situations - so two columns are provided in the table.

Procedure for Using z Tables to Calculate the Range for μ

Once you've located the z score, you can calculate the predicted range for μ using the general formula below

$$\text{Predicted range for } \mu = \bar{x} \pm \frac{s_x}{\sqrt{n}} z$$

where $\bar{x}$ is your *sample mean*.

$s_{\bar{x}}$ is the *sample* standard deviation

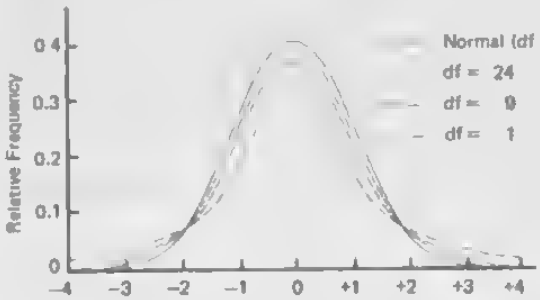
Note : For large samples the sample standard deviation (s_x) turns out to be very nearly equal to the *population standard deviation* (usually labeled σ). The formula for the range is always correct when written with σ in place of s_x - and you'll see it written that way quite often in textbooks. z is the "z score" for the degree of certainty you select.

Remember that this particular technique works only for large samples taken from larger populations. (Again the boundary line for large samples is usually taken to be 30 elements, and a large population is considered to be 100 elements or more).

Analysing with Small Samples : t Scores

By now the question may have arisen - "What happens when I don't have 30 samples - let's say I've got only 5 or 10 ? What do I do then ? " Statisticians have been busily at work on this problem, too. It turns out that as the number of samples goes below 30, the "normal" curve can no longer be accurately used to describe the distribution of the sample means. Statisticians have found a different family of curves that *does work* - if the population is nearly normally distributed - called t curves.

The shape of any t curve depends on what's called the number of *degrees of freedom* (df) for your particular sample. The number of degrees of freedom in most cases is considered to be equal to the *number of elements in your sample minus one* (df = n − 1). The shapes of various t curves are shown in the figure below. Note for a very large number of degrees of freedom (essentially df = 31 or greater), the t curve *becomes* the normal curve (and z scores can be used).



Areas under the t curves have also been tabulated for you in t "score" tables (*Tables B* and *C* in the *Appendix* are t score tables for your use). With t scores you can analyse small sample data in much the same way large sample data is analysed with z scores. Here's the step-by step procedure to follow.

With the aid of your calculator, calculate the sample mean ($\bar{x}$) and sample standard deviation ($s_{\bar{x}}$).

With this information, you'll be calculating a *predicted* range for the population mean. Decide how certain you want (or need) to be that the population mean will be in your predicted range. For this level of certainty look up the appropriate t score in *Table B* or *C* in the *Appendix*. (Use *Table B* if your decision involves *only* a maximum or minimum value for μ; otherwise use *Table C*.) The value for df you use (the degrees of freedom) is the *number of elements in your sample minus one* (n − 1).

Once you've located the t score, the predicted range for the population mean can be calculated using the formula :

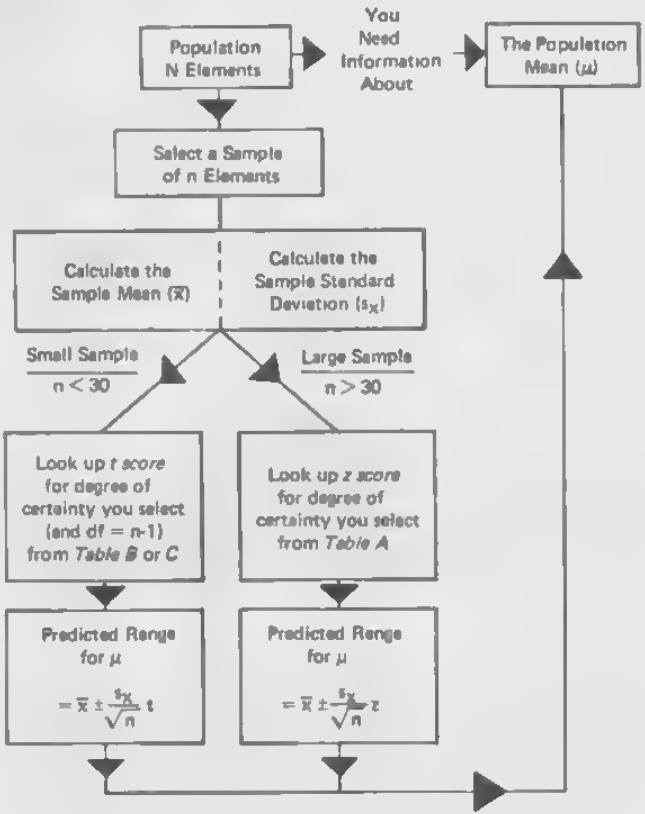
$$\text{Predicted range for the population mean} = \bar{x} \pm \frac{s_{\bar{x}}}{\sqrt{n}} t$$

Summary on Statistical Inference

So basically one process of statistical inference, that can be of great use to you in decision making, involves taking data from a sample - and from that calculating a predicted range for the population mean. This range will tell you, to the degree of certainty you select, where the population mean lies. In the chapter on "testing claims" we discuss how you'd compare this predicted range for the population mean value claimed by a manufacturer or supplier for a given product, part, etc. If the *claimed* mean value *does not* fall in the range you predict - there may be a problem ! As discussed in the examples, you may want to *reject* a shipment or talk further with your supplier at that point. You'll see several other examples that involve this application of statistical inference. Armed with data from a sample, you can make more accurate decisions about claims being made for a population - using your calculator and a little statistical maths.

The steps involved in the process of statistical inference can be summarised in the following diagram

Steps In Analysing *Sample* Data, to Calculate the *Predicted Range for the Population Mean* :



One Further Note on Standard Deviation

Statisticians use two different formulae for calculating standard deviation (in their continued effort to be as accurate as possible). When calculating the *standard deviation of a population* (σ) they use the formula .

$$\sigma = \sqrt{\frac{\sum_i (x_i - \bar{x})^2}{N}}$$

where N is the number of elements in the population.

Note : The symbol $\sum_i$ used here (and elsewhere) just means "the sum of" In this case $\sum_i (x_i - \bar{x})^2$ means to add all the values of $(x_i - \bar{x})^2$ for i going from 1 to N

On your calculator, when you're handling an *entire population*, you can enter your data with the $\Sigma+$ key, and calculate :
- the population mean (μ) by pressing 2nd MEAN
- the standard deviation of the population (σ) by pressing 2nd VAR $\sqrt{x}$

When calculating the *standard deviation of a sample*, (s_x), the formula used is :

$$s_x = \sqrt{\frac{\sum_i (x_i - \bar{x})^2}{n - 1}}$$

where n is the number of elements in the sample.

On your calculator, when handling a *sample*, you can enter your data with the $\Sigma+$ key, and calculate
- the sample mean ($\bar{x}$) by pressing 2nd MEAN
- the sample standard deviation (s_x) by pressing 2nd SDEV

The complete reasoning behind the difference in these two calculations is beyond the scope of this book. One consideration is that using n − 1 rather than n in the denominator of the sample standard deviation results in a value for s_x that is a *better estimate* for the standard deviation of the population from which the sample is taken. (For values of $n > 30$, the difference between these two calculations becomes very small).

Calculator Decision Making

This chapter has surveyed quite a bit of information, quite briefly. In the application chapters of this book we boil down these facts to step-by-step, easy to apply procedures - along with keystroke sequences and sample calculations. *Using* these powerful methods (particularly with your calculator helping out) is really quite easy, and you needn't have studied statistics for years to do it. Once you see the methods "in action" in application situations - many of the procedures we've outlined in this chapter become clearer - and you'll see more clearly how they can help you in calculating better decisions.

We've tried to keep the number of "symbols" used in this book to a minimum - we've introduced and discussed many of them in this chapter - and they're tabulated for you below (A complete table of all symbols used is also included in a later *Appendix* for your convenience)

	Population	Sample	Calculator Key Sequence
Number of Elements	N	n	Enter Value of Element, Press $\boxed{\Sigma+}$
Mean	μ	$\bar{x}$	Press $\boxed{2nd}$ $\boxed{Mean}$
Standard Deviation	$\sigma = \sqrt{\frac{\sum (x_i - \bar{x})^2}{N}}$	$s_x = \sqrt{\frac{\sum (x_i - \bar{x})^2}{n - 1}}$	σ_x Press $\boxed{2nd}$ $\boxed{\sigma_x}$ $\boxed{\sqrt{\ }}$ s_x Press $\boxed{2nd}$ $\boxed{s_x}$

$s_{\bar{x}}$ = Standard Deviation of the Sample Means

z = z score for a selected degree of certainty

t = t score for a selected degree of certainty and specific number of degrees of freedom (df)

Summary of Symbols

df — degrees of freedom
F — F number from *Table D* or *E*
n — number of elements in a sample
N — number of elements in a population
r — correlation coefficient
r_{test} — test correlation coefficient
Sx_h — standard deviation of the "high" sample
Sx_l — standard deviation of the "low" sample
s_x — standard deviation of a sample
$s_{\overline{x}}$ — standard deviation of a sample means
σ — standard deviation of a population
t — t number from *Table B* or *C*
x_i — the ith element of a sample or population
$\overline{x}$ — sample mean
μ — population mean
z — z number from *Table A*

Table A

Degree of Certainty	z Scores * Column I For Checking Only an Upper or Lower Level	Column II For Checking Both an Upper and Lower Level
60	0.26	0.84
65	0.39	0.94
70	0.53	1.04
75	0.68	1.15
80	0.84	1.28
85	1.04	1.44
90	1.28	1.65
95	1.65	1.96
99	2.33	2.58

*z scores are often called "z values" in this book.

Table 6 **t Scores***

(For Checking Only an Upper or a Lower Limit)

Degrees of Freedom (df)	Level of Certainty			
	90%	95%	99%	99.5%
1	3 078	6 314	31 821	63 657
2	1 886	2 920	6 965	9 925
3	1 638	2 353	4 541	5 841
4	1 533	2 132	3 747	4 604
5	1 476	2 015	3 365	4 032
6	1 440	1 943	3 143	3 707
7	1 415	1 895	2 998	3 499
8	1 397	1 860	2 896	3 355
9	1 383	1 833	2 821	3 250
10	1 372	1 812	2 764	3 169
11	1 363	1 796	2 718	3 106
12	1 356	1 782	2 681	3 055
13	1 350	1 771	2 650	3 012
14	1 345	1 761	2 624	2 977
15	1 341	1 753	2 602	2 947
16	1 337	1 746	2 583	2 921
17	1 333	1 740	2 567	2 898
18	1 330	1 734	2 552	2 878
19	1 328	1 729	2 539	2 861
20	1 325	1 725	2 528	2 845
21	1 323	1 721	2 518	2 831
22	1 321	1 717	2 508	2 819
23	1 319	1 714	2 500	2 807
24	1 318	1 711	2 492	2 797
25	1 316	1 708	2 485	2 787
26	1 315	1 706	2 479	2 779
27	1 314	1 703	2 473	2 771
28	1 313	1 701	2 467	2 763
29	1 311	1 699	2 462	2 756
30	1 310	1 697	2 457	2 750
40	1 303	1 684	2 423	2 704
60	1 296	1 671	2 390	2 660
120	1 289	1 658	2 358	2 617
∞	1 282	1 645	2 326	2 576

*t scores are often called "t values" in this book.

Table C

t Scores

(For Checking Both Upper and Lower Limits)

Degrees of Freedom (df)	Level of Certainty				
	80%	90%	95%	99%	99.9%
1	3.078	6.314	12.706	63.657	636.619
2	1.886	2.920	4.303	9.925	31.598
3	1.638	2.353	3.182	5.841	12.941
4	1.533	2.132	2.776	4.604	8.610
5	1.476	2.015	2.571	4.032	6.859
6	1.440	1.943	2.447	3.707	5.959
7	1.415	1.895	2.365	3.499	5.405
8	1.397	1.860	2.306	3.355	5.041
9	1.383	1.833	2.262	3.250	4.781
10	1.372	1.812	2.228	3.169	4.587
11	1.363	1.796	2.201	3.106	4.437
12	1.356	1.782	2.179	3.055	4.318
13	1.350	1.771	2.160	3.012	4.221
14	1.345	1.761	2.145	2.977	4.140
15	1.341	1.753	2.131	2.947	4.073
16	1.337	1.746	2.120	2.921	4.015
17	1.333	1.740	2.110	2.898	3.965
18	1.330	1.734	2.101	2.878	3.922
19	1.328	1.729	2.093	2.861	3.883
20	1.325	1.725	2.086	2.845	3.850
21	1.323	1.721	2.080	2.831	3.819
22	1.321	1.717	2.074	2.819	3.792
23	1.319	1.714	2.069	2.807	3.767
24	1.318	1.711	2.064	2.797	3.745
25	1.316	1.708	2.060	2.787	3.725
26	1.315	1.706	2.058	2.779	3.707
27	1.314	1.703	2.052	2.771	3.690
28	1.313	1.701	2.048	2.763	3.674
29	1.311	1.699	2.045	2.756	3.659
30	1.310	1.697	2.042	2.750	3.646
40	1.303	1.684	2.021	2.704	3.551
60	1.296	1.671	2.000	2.660	3.460
120	1.289	1.658	1.980	2.617	3.373
∞	1.282	1.645	1.960	2.576	3.291

Table D

APPENDIX
(Values of F for 95 % Level of Certainty)

Degrees of Freedom of the Numerator (columns); Degrees of Freedom of the Denominator (rows)

	1	2	3	4	5	6	7	8	9	10	12	15	20	30	60	120	∞
1	161.4	199.5	215.7	224.6	230.2	234.0	236.8	238.9	240.5	241.9	243.9	245.9	248.0	250.1	252.2	253.3	254.3
2	18.51	19.00	19.16	19.25	19.30	19.33	19.35	19.37	19.38	19.40	19.41	19.43	19.45	19.46	19.48	19.49	19.50
3	10.13	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81	8.79	8.74	8.70	8.66	8.62	8.57	8.55	8.53
4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00	5.96	5.91	5.86	5.80	5.75	5.69	5.66	5.63
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77	4.74	4.68	4.62	4.56	4.50	4.43	4.40	4.36
6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10	4.06	4.00	3.94	3.87	3.81	3.74	3.70	3.67
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68	3.64	3.57	3.51	3.44	3.38	3.30	3.27	3.23
8	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39	3.35	3.28	3.22	3.15	3.08	3.01	2.97	2.93
9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18	3.14	3.07	3.01	2.94	2.86	2.79	2.75	2.71
10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.98	2.91	2.85	2.77	2.70	2.62	2.58	2.54
11	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90	2.85	2.79	2.72	2.65	2.57	2.49	2.45	2.40
12	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80	2.75	2.69	2.62	2.54	2.47	2.38	2.34	2.30
13	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71	2.67	2.60	2.53	2.46	2.38	2.30	2.25	2.21
14	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65	2.60	2.53	2.46	2.39	2.31	2.22	2.18	2.13
15	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59	2.54	2.48	2.40	2.33	2.25	2.16	2.11	2.07
16	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54	2.49	2.42	2.35	2.28	2.19	2.11	2.06	2.01
17	4.45	3.59	3.20	2.96	2.81	2.70	2.61	2.55	2.49	2.45	2.38	2.31	2.23	2.15	2.06	2.01	1.96
18	4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.46	2.41	2.34	2.27	2.19	2.11	2.02	1.97	1.92
19	4.38	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.42	2.38	2.31	2.23	2.16	2.07	1.98	1.93	1.88
20	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39	2.35	2.28	2.20	2.12	2.04	1.95	1.90	1.84
21	4.32	3.47	3.07	2.84	2.68	2.57	2.49	2.42	2.37	2.32	2.25	2.18	2.10	2.01	1.92	1.87	1.81
22	4.30	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.34	2.30	2.23	2.15	2.07	1.98	1.89	1.84	1.78
23	4.28	3.42	3.03	2.80	2.64	2.53	2.44	2.37	2.32	2.27	2.20	2.13	2.05	1.96	1.86	1.81	1.76
24	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30	2.25	2.18	2.11	2.03	1.94	1.84	1.79	1.73
25	4.24	3.39	2.99	2.76	2.60	2.49	2.40	2.34	2.28	2.24	2.16	2.09	2.01	1.92	1.82	1.77	1.71
26	4.23	3.37	2.98	2.74	2.59	2.47	2.39	2.32	2.27	2.22	2.15	2.07	1.99	1.90	1.80	1.75	1.69
27	4.21	3.35	2.96	2.73	2.57	2.46	2.37	2.31	2.25	2.20	2.13	2.06	1.97	1.88	1.79	1.73	1.67
28	4.20	3.34	2.95	2.71	2.56	2.45	2.36	2.29	2.24	2.19	2.12	2.04	1.96	1.87	1.77	1.71	1.65
29	4.18	3.33	2.93	2.70	2.55	2.43	2.35	2.28	2.22	2.18	2.10	2.03	1.94	1.85	1.75	1.70	1.64
30	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21	2.16	2.09	2.01	1.93	1.84	1.74	1.68	1.62
40	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12	2.08	2.00	1.92	1.84	1.74	1.64	1.58	1.51
60	4.00	3.15	2.76	2.53	2.37	2.25	2.17	2.10	2.04	1.99	1.92	1.84	1.75	1.65	1.53	1.47	1.39
120	3.92	3.07	2.68	2.45	2.29	2.17	2.09	2.02	1.96	1.91	1.83	1.75	1.66	1.55	1.43	1.35	1.25
∞	3.84	3.00	2.60	2.37	2.21	2.10	2.01	1.94	1.88	1.83	1.75	1.67	1.57	1.46	1.32	1.22	1.00

Table E

APPENDIX

(Values of F for 99 % Level of Certainty)

Degrees of Freedom of the Numerator (columns); Degrees of Freedom of the Denominator (rows)

	1	2	3	4	5	6	7	8	9	10	12	15	20	30	60	120	∞
1	161.4	199.5	215.7	224.6	230.2	234.0	236.8	238.9	240.5	241.9	243.9	245.9	248.0	250.1	252.2	253.3	254.3
2	18.51	19.00	19.16	19.25	19.30	19.33	19.35	19.37	19.38	19.40	19.41	19.43	19.45	19.46	19.48	19.49	19.50
3	10.13	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81	8.79	8.74	8.70	8.66	8.62	8.57	8.55	8.53
4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00	5.96	5.91	5.86	5.80	5.75	5.69	5.66	5.63
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77	4.74	4.68	4.62	4.56	4.50	4.43	4.40	4.36
6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10	4.06	4.00	3.94	3.87	3.81	3.74	3.70	3.67
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68	3.64	3.57	3.51	3.44	3.38	3.30	3.27	3.23
8	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39	3.35	3.28	3.22	3.15	3.08	3.01	2.97	2.93
9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18	3.14	3.07	3.01	2.94	2.86	2.79	2.75	2.71
10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.98	2.91	2.85	2.77	2.70	2.62	2.58	2.54
11	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90	2.85	2.79	2.72	2.65	2.57	2.49	2.45	2.40
12	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80	2.75	2.69	2.62	2.54	2.47	2.38	2.34	2.30
13	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71	2.67	2.60	2.53	2.46	2.38	2.30	2.25	2.21
14	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65	2.60	2.53	2.46	2.39	2.31	2.22	2.18	2.13
15	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59	2.54	2.48	2.40	2.33	2.25	2.16	2.11	2.07
16	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54	2.49	2.42	2.35	2.28	2.19	2.11	2.06	2.01
17	4.45	3.59	3.20	2.96	2.81	2.70	2.61	2.55	2.49	2.45	2.38	2.31	2.23	2.15	2.06	2.01	1.96
18	4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.46	2.41	2.34	2.27	2.19	2.11	2.02	1.97	1.92
19	4.38	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.42	2.38	2.31	2.23	2.16	2.07	1.98	1.93	1.88
20	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39	2.35	2.28	2.20	2.12	2.04	1.95	1.90	1.84
21	4.32	3.47	3.07	2.84	2.68	2.57	2.49	2.42	2.37	2.32	2.25	2.18	2.10	2.01	1.92	1.87	1.81
22	4.30	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.34	2.30	2.23	2.15	2.07	1.98	1.89	1.84	1.78
23	4.28	3.42	3.03	2.80	2.64	2.53	2.44	2.37	2.32	2.27	2.20	2.13	2.05	1.96	1.86	1.81	1.76
24	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30	2.25	2.18	2.11	2.03	1.94	1.84	1.79	1.73
25	4.24	3.39	2.99	2.76	2.60	2.49	2.40	2.34	2.28	2.24	2.16	2.09	2.01	1.92	1.82	1.77	1.71
26	4.23	3.37	2.98	2.74	2.59	2.47	2.39	2.32	2.27	2.22	2.15	2.07	1.99	1.90	1.80	1.75	1.69
27	4.21	3.35	2.96	2.73	2.57	2.46	2.37	2.31	2.25	2.20	2.13	2.06	1.97	1.88	1.79	1.73	1.67
28	4.20	3.34	2.95	2.71	2.56	2.45	2.36	2.29	2.24	2.19	2.12	2.04	1.96	1.87	1.77	1.71	1.65
29	4.18	3.33	2.93	2.70	2.55	2.43	2.35	2.28	2.22	2.18	2.10	2.03	1.94	1.85	1.75	1.70	1.64
30	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21	2.16	2.09	2.01	1.93	1.84	1.74	1.68	1.62
40	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12	2.08	2.00	1.92	1.84	1.74	1.64	1.58	1.51
60	4.00	3.15	2.76	2.53	2.37	2.25	2.17	2.10	2.04	1.99	1.92	1.84	1.75	1.65	1.53	1.47	1.39
120	3.92	3.07	2.68	2.45	2.29	2.17	2.09	2.02	1.96	1.91	1.83	1.75	1.66	1.55	1.43	1.35	1.25
∞	3.84	3.00	2.60	2.37	2.21	2.10	2.01	1.94	1.88	1.83	1.75	1.67	1.57	1.46	1.32	1.22	1.00

Anthony, Robert N & Glen A Welsch, *Fundamentals of Management Accounting.* Homewood, Illinois Richard D Irwin, 1974

Anthony, Robert N & James S Reece, *Management Accounting Text and Cases* Fifth edition, Homewood, Illinois Richard D. Irwin, 1975.

Chou, Ya-lun, *Statistical Analysis.* New York Holt, Rinehart and Winston, 1975.

Ferguson, George A , *Statistical Analysis in Psychology and Education,* New York McGraw-Hill Book Company, 1966

Freund, John E., *Mathematical Statistics.* Englewood Cliffs, New Jersey Prentice-Hall, Inc., 1962

Horngren, Charles T., *Accounting for Management Control,* Englewood Cliffs New Jersey Prentice-Hall, Inc , 1974

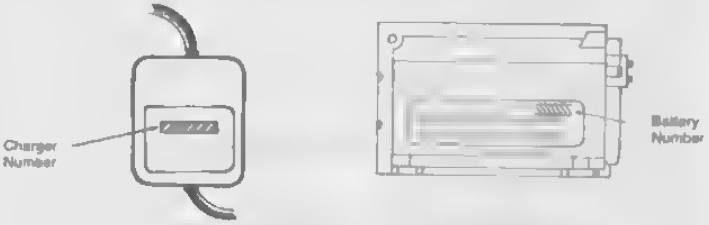
Horngren, Charles T , *Cost Accounting A Managerial Emphasis.* Englewood Cliffs, New Jersey Prentice-Hall, Inc., 1972

Hummel, Paul M & Charles Seebeck, *Mathematics of Finance,* New York McGraw-Hill Book Company, 1971

Weston, J. Fred & Eugene F. Brigham, *Essentials of Managerial Finance* Third edition, Hinsdale, Illinois : The Dryden Press, 1974

Normal Operation

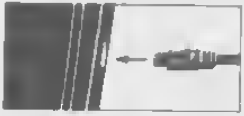
Your calculator is designed for portable operation with periodic recharging of the battery pack with the adapter/charger supplied. It is important that the proper adapter/charger is used. If replacement of the battery pack or charger becomes necessary, be sure that an exact replacement is obtained.



Your calculator uses the BP7 with the AC9900R adapter/charger

Caution : Use of other than the proper Adapter/Charger may apply improper voltage to your calculator and damage the unit.

To ensure maximum portable operation time, connect the Adapter/Charger to a standard 220V/50 Hz outlet, plug into calculator, and charge battery pack at least 4 hours with the calculator OFF or 10 hours with the calculator ON The adapter/charger and battery pack may become warm when used on AC power This is normal and of no consequence.



When the battery pack is fully charged, the calculator will operate approximately 2 to 3 hours before recharging is necessary However, don't hesitate to connect the adapter/charger if you know or suspect the battery pack is nearly discharged A battery pack near discharge can adversely effect all calculator operations, giving erroneous results A discharged battery pack is typically indicated by a dim, erratic or blank display.

While individual cell life in a battery pack is difficult to predict, under normal use, rechargeable batteries have a life of 2 to 3 years or about 500 to 1000 recharge cycles.

Periodic Recharging

Although the calculator will operate indefinitely with the adapter/charger connected, the rechargeable battery pack can lose its storage capacity if it is not allowed to discharge occasionally. For maximum battery life, it is recommended that you operate the calculator as a portable at least twice a month, allowing the batteries to discharge, then recharge accordingly.

Excessive Battery Discharging

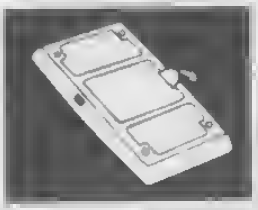
If the calculator is left on for an extended period of time after the battery pack is discharged (accidentally left on overnight, for example), connect the adapter/charger for at least 24 hours with the calculator OFF. If this does not restore normal battery operation, the battery pack should be replaced. Repeated occurences of excessive battery discharging will permanently damage the battery pack.

Storage

If the calculator is stored or unused for several weeks, the battery pack will probably need recharging before portable use. The battery pack will not leak corrosive material; therefore, it is safe to store the calculator with the battery pack installed.

Battery Pack Replacement

The battery pack can be quickly and simply removed from the calculator. Hold the calculator with keys facing down. Place a small coin in the slot in the bottom of the calculator. A slight prying motion with the coin will pop the slotted end of the pack out of the calculator. Carefully disconnect the wires that attach the battery pack to the calculator. The pack can then be removed entirely from the calculator.



The metal contacts on the battery pack (where charger and calculator plug in) are the battery terminals. Care should always be taken to prevent any metal object from coming into contact with these terminals and shorting the batteries.

To re-insert the battery pack, first attach the connecting wires to the terminals of the battery pack. Alignment should not be a problem as the connector will only fit in one position. Then, place the pack into the compartment so that the small step on the end of the pack fits under the edge of the calculator bottom. A small amount of pressure on the battery pack will snap it properly into position. (Do not force. It will fit easily when properly oriented).

In Case of Difficulty

In the event that you have difficulty with your calculator, the following instructions will help you to analyse the problem. You may be able to correct your calculator problem without returning the unit to a service facility.

Symptom	
1. Display is blank for no obvious reason.	Press and hold R/S momentarily. If display returns, the calculator was running a long program or operating in a continuous program loop. The battery pack may be discharged or improperly installed. Also, check to be sure the ON-OFF switch is fully in the ON position.
2. Display shows erroneous results, flashes erratic numbers, grows dim, or goes blank.	The battery pack is probably discharged or improperly connected. Refer to *Normal Operation.*
3. Display flashes while performing keyboard operations.	An invalid operation or key sequence has been pressed or the limits of the calculator have been violated. See Appendix B for a list of these conditions.

If none of the above procedures corrects the difficulty, return the calculator and charger PREPAID and INSURED to the applicable Service Facility listed on the back cover. Send a brief description of the problem you found and do not forget to give a clear indication of your name and address. The shipment should be carefully packaged and adequately protected against shock and rough handling. Do not forget to attach a proof-of-purchase date (sales receipt, invoice, attached coupon). Keep the original, only send a copy. Units returned without proof-of-purchase date will be repaired at the services rates in effect at the time of return. You will find our repair centre addresses on the back-cover.

If the calculator is out of warranty, service rates in effect at time of return will be charged. Please include information on the difficulty experienced with the calculator as well as return address information including name, address, city, state and post code. The shipment should be carefully packaged, and adequately protected against shock and rough handling.

Suggestions

Because of the number of suggestions which come to Texas Instruments from many sources, containing both new and old ideas, Texas Instruments will consider such suggestions only if they are freely given to Texas Instruments. It is the policy of Texas Instruments to refuse to receive any suggestions in confidence. Therefore, if you wish to share your suggestion with Texas Instruments, or if you wish us to review any calculator program key sequence which you have developed, please include the following in your letter:

> "All of the information forwarded herewith is presented to Texas Instruments on a nonconfidential, nonobligatory basis; no relationship, confidential or otherwise, expressed or implied, is established with Texas Instruments, by this presentation. Texas Instruments may use, copyright, distribute, publish, reproduce, or dispose of the information in any way without compensation to me".

A flashing display indicates that the internal limits of the calculator have been violated or that an invalid calculator operation has been requested. Pressing [CE] [CLR] or [2nd] [CA] stops the flashing. [CLR] or [2nd] [CA] also clears the display and pending operations, [CE] stops the flashing only, permitting further calculations with undisturbed pending operations. The display will flash for the following reasons:

1. Calculation entry or result (in display or memories) outside the range of the calculator, $\pm 1 \times 10^{-99}$ to $\pm 9.9999999 \times 10^{99}$. Some entries or results smaller than 10^{-97} or larger than 10^{99} can cause an internal underflow/overflow condition which results in a flashing display.
2. Inverse of a trigonometric or hyperbolic function with an invalid value for the argument, such as $\sin^{-1} x$ with $|x|$ greater than 1.
3. Root or logarithm of a negative number.
4. Raising of a negative number to any power.
5. Pressing two operation keys in succession. This affects +, −, ×, ÷, y^x, $\sqrt[x]{y}$ or Δ %.
6. Pressing [=] or [)] after +, −, ×, ÷, y^x, $\sqrt[x]{y}$ or Δ %.
7. Having more than 9 open parentheses or more than 4 pending operations. The 10th parenthesis or 5th operation is not accepted so processing can be continued after pressing [CE].
8. Dividing a number by zero.
9. Factorial of any number except a non-negative integer ≤ 69.
10. Any memory operation that is not followed by 0→9, [CLR] or [2nd] [CA].
11. An x or y value outside the range $\pm 10^{\pm 99}$ in Rectangular to Polar Conversions.
12. In linear regression calculations, if the line parallels the y-axis, attempting to calculate slope, intercept, correlation, x' or y' will cause flashing. If the line parallels the x-axis, the display flashes when attempting to calculate x' or correlation.
13. Calculation of slope, intercept, correlation, y', x' or standard deviation with less than 2 data points entered.
14. 0^x and $\sqrt[x]{0}$ produces flashing overflow.
15. Key sequence x_1 [2nd] [Σ] x_2 [=] where $x_2 = 0$.
16. Arguments that do not satisfy the following limits cause a flashing display.

Function	Limit
$\sin^{-1} x$, $\cos^{-1} x$	$-1 \leq x \leq 1$
sinh x, cosh x	$0 \leq \lvert x \rvert \leq 227.95592$
ln x	$1 \times 10^{-99} \leq x < 1 \times 10^{100}$
log x	$1 \times 10^{-99} \leq x < 1 \times 10^{100}$
$\sinh^{-1} x$	$-10^{50} < x < 10^{50}$
$\cosh^{-1} x$	$1 \leq x < 10^{50}$
$\tanh^{-1} x$	$0 \leq \lvert x \rvert < 1.0$
e^x	$-227.95592 \leq x \leq 230.25850$
10^x	$-99 < x < 100$
x!	$0 \leq x \leq 69$ (integer)

Rounding and Guard Digits

Calculators like all other electrical and mechanical devices, must operate with a fixed set of rules within preset limits.

The basic mathematical tolerance of the calculator is controlled by the number of digits it uses for calculations. The calculator appears to use 8 digits as shown by the display, but actually uses 11 digits to perform all calculations. Combined with the built-in 5/4 rounding capability, these extra digits guard the eight digit display to improve accuracy. Consider the following example in the absence of these guard digits.

$$1/3 \times 3 = 0.9999999 \text{ (inaccurate)}$$

The example shows that $1 \div 3 = 0.3333333$ when multiplied by 3 and produces an inaccurate answer. An eleven-digit string of nines would *round* to 1.

The higher order mathematical functions use iterative calculations. The three guard digits normally allow the accuracy of higher order functions to be better than or equal to ± 1 in the last displayed digit.

Normally, there is no need to even consider these guard digits. On certain calculations however, the guard digits may appear as an answer when not expected. The mathematical limits of finite operation - word length, truncation and rounding errors do not allow the guard digits to always be completely accurate. Therefore, when subtracting two functions which are mathematically equal, the calculator may display a non-zero result.
For example, the difference in results from solving $116)^2$ using [x^2] and [y^x] is 3×10^{-5}.

Mathematical Limits - There are a few instances in the iterative solution of higher order functions where display accuracy begins to deteriorate as the function approaches a discontinuous or undefined point. For example, the tangent of 89 degrees is accurate for all displayed digits. However, the tangent of 89.99999 is accurate to only four places. Another example is when the y^x function has a y value that approaches 1 and the x value is very large. The displayed result for 0.999^{-100} is accurate for all displayed digits, where $0.999^{-160000}$ is only accurate to five places.

Trigonometric Angle Conversions - When using trigonometric functions with angles greater than ± 360 degrees, + 2π radians or ± 400 grads at multiples of 90 degrees, etc., the results may appear as very small values in scientific notation. Examples are sin 3600 = 2×10^{-9} and sin 36000 = 2×10^{-5}. These nonzero results are due to slight guard digit inaccuracies developed when the large angle is converted back to a first revolution angle.

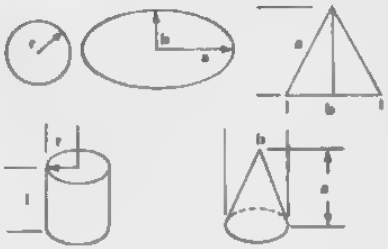
In the learn mode, the display shows you where the program counter is positioned and the instruction presently in that location. The instruction is represented by a two-digit code that comes directly from that key's location on the keyboard.

The table below illustrates the key codes for each function.

PROGRAM KEY CODES

Key	Code	Key	Code	Key	Code	Key	Code	Key	Code
	None	[illegible]	17	[illegible]	15	[illegible]	16	CA	10
2nd	None	sin	12	cos	13	tan	14	CLR	15
	26	[illegible]	27	log	28	10^x	25	x!	20
INV	21	[illegible]	22	lnx	23	e^x	24	$\sqrt[x]{y}$	25
P-R	36	MEAN	37	S.DEV	38	VAR	39	CORR	30
x⇄y	31	x^2	32	$\sqrt{x}$	33	1/x	34	y^x	35
Σ−	46	ENG	47	[illegible]	48	π	46	SLOPE	40
Σ+	41	EE	42	(	43	)	44	÷	45
FIX	50	Deg	57	Rad	58	Grad	59	INTCP	50
STO	51	7	07	8	08	9	09	×	55
EXC	06	[illegible]	67	[illegible]	68	[illegible]	59	x'	60
RCL	61	4	04	5	05	6	06	−	65
PROD	76	F-C	77	D-R	75	[illegible]	79	y'	70
SUM	71	1	01	2	02	3	03	+	75
	86		87	LRN	None	SST	None	[illegible]	80
CE	81	0	00	•	83	+/−	84	=	85

Through normal usage you will become familiar with the more common instruction codes so that constant reference to this table will not be necessary. The others are quickly determined by counting row and column numbers on the keyboard itself.



Circumference	.	Circle	$2\pi r$
Area	:	Circle	πr^2
		Ellipse	πab
		Sphere	$4\pi r^2$
		Cylinder	$2\pi r\,[r + l]$
		Triangle	$\frac{1}{2}\,ab$
Volume	:	Ellipsoid of revolution about "a" axis	$4/3\pi b^2 a$
		Ellipsoid of revolution about "b" axis	$4/3\pi a^2 b$
		Sphere	$4/3\pi r^3$
		Cylinder	$\pi r^2 l$
		Cone	$\frac{\pi b^2 a}{12}$
Analytical	:	Circle	$\frac{x^2}{r^2} + \frac{y^2}{r^2} = 1$
		Ellipse	$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$
		Hyperbola	$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$
		Parabola	$y^2 = \pm 2px$
		Line	$y = mx + b$

Trigonometric Relations

$$\sin\Theta = \frac{y}{r}$$

$$\cos\Theta = \frac{x}{r}$$

$$\tan\Theta = \frac{y}{x}$$

$$\sin^2\Theta + \cos^2\Theta = 1$$

$$e^{i\Theta} = \cos\Theta + i\sin\Theta \qquad i = \sqrt{-1}$$

Law of Cosines

$$a^2 + b^2 - 2ab\cos\Theta = c^2$$

Law of Exponents

$$a^x \times a^y = a^{x+y} \qquad \frac{1}{a^x} = a^{-x}$$

$$(ab)^x = a^x \times b^x \qquad \frac{a^x}{a^y} = a^{x-y}$$

$$(a^x)^y = a^{xy} \qquad a^0 = 1$$

Laws of Logarithms

$$\ln(y^x) = x\ln y$$

$$\ln(ab) = \ln a + \ln b$$

$$\ln\left(\frac{a}{b}\right) = \ln a - \ln b$$

ONE YEAR-LIMITED WARRANTY

The TI 51-III electronic calculator (including charger) from Texas Instruments is warranted to the original purchaser for a period of one (1) year from the original purchase date – under normal use and service – against defective materials or workmanship

This warranty is void if the calculator has been damaged by accident or unreasonable use, neglect, improper service or other causes not arising out of defects in material or workmanship.

TEXAS INSTRUMENTS SHALL NOT BE LIABLE FOR LOSS OF USE OF THE CALCULATOR OR OTHER INCIDENTAL OR CONSEQUENTIAL COSTS, EXPENSES OR DAMAGES INCURRED BY THE PURCHASER.

During the above one-year period, the calculator or its defective parts will be repaired, adjusted and/or replaced with a reconditioned model of equivalent quality ("REFURBISHED") at manufacturer's option without charge to the purchaser when the calculator is returned, prepaid and insured, with proof-of-purchase date, to Texas Instruments **UNITS RETURNED WITHOUT PROOF-OF-PURCHASE DATE WILL BE REPAIRED AT THE SERVICE RATES IN EFFECT AT THE TIME OF RETURN**

In the event of replacement with a reconditioned model, the replacement unit will continue the warranty of the original calculator product or 90 days whichever is longer

THIS WARRANTY OFFERS YOU SPECIFIC LEGAL RIGHTS AND DOES NOT AFFECT ANY STATUTORY CONSUMER RIGHTS.

IMPORTANT : Before returning your calculator for repair, carefully review service and mailing instructions in this manual

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